

APLUM

REFERENCE

MANUAL

By

Clark Wiedmann

Second Edition

APLUM Reference Manual

by
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PREFACE

This manual describes a version of the programming language APL developed by the University Computing Center of the University of Massachusetts at Amherst. The system described here, known as APLUM, was developed under the direction of James H. Burrill, who was responsible for the overall system design, planning, and coordination, and who wrote a considerable portion of the system. Other programmers on the project included Rick Mayforth, who was responsible for system commands, system functions, and system variables; Sheldon Gersten, who did work on mixed functions and shared variables; Brian Arnold, who worked primarily on output, format functions, and shared variables; Jeff Dean, who worked on mixed functions, workspace conversion and system functions; Judy Smith, who wrote inner product; and Clark Wiedmann, who worked on function definition, the file system, and mixed functions. In addition, Pat Driscoll and Wendy Mayfield assisted with documentation. The Raytheon Corporation provided assistance with many of the scalar functions, as did the Canadian Development Division of Control Data Corporation. The project was supported in part by a grant from Control Data Corporation. Special thanks are due to the entire staff of the University Computing Center for their assistance with the APL project. Credit is also due to the developers of several other versions of the APL language for the features and careful design that guided us in the development of APLUM.

Primary objectives of the design for APLUM were: to achieve a very high level of performance on the CDC 5600 and CYBER computers under the KRONOS operating system, to provide a flexible file system, to incorporate system functions and variables, to provide all system command capabilities to user-defined functions, to allow communication with programs in other languages, and to allow all workspace areas (including the symbol table and file buffers) to change size dynamically according to changing needs. The system was also designed with a storage management scheme that will enable future implementation

of arrays of arrays or future extensions to allow functions and variables not in use to reside on secondary memory devices, thus allowing an almost unlimited workspace size.

This edition is intended to describe version 2.12 of APLUM. For information about recent changes, see the APLNEWS workspace on the system. This is accessed by typing the APL command [LOAD '*APL1 APLNEWS'.

This book is arranged as a reference manual and not as a teaching manual. The intent is to accurately describe particular details of APLUM, but not to teach APL to the novice. Hence the reader will find that this book lacks the wealth of examples, problems, and exercises that are usually found in a teaching manual. Some previous knowledge of APL is almost essential in order to make use of this manual, and the following books are recommended as possible introductions to the language:

APL 360 Primer, Paul Berry, IBM Technical Publications, New York, 1969. Covers in detail function definition and many primitive functions and system commands, but barely touches on matrices, higher-order arrays, and many mixed functions. Recommended for the occasional user of APL.

Handbook of APL Programming, Clark Wiedmann, Petrocelli Books, N.Y., 1974. A comprehensive introduction for the more advanced user of APL who will make extensive use of the capabilities available.

Although this reference manual is intended for the reader who already has some knowledge of APL, it is recognized that all too often programmers are introduced to a new language through a reference manual. Consequently, a short introduction has been provided so that the following chapters will make some sense to the APL novice. The introduction also attempts to emphasize some of the more important features of the language to an extent that the organization of later chapters does not allow.

Clark Wiedmann

Amherst, 1975

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Introduction. A Sample Terminal Session

This short introduction to APL shows a sample terminal session from the time of signing on until the time of signing off. This section attempts to emphasize some of the important facilities of APL, and attempts to show the dynamic nature of APL (which may not be evident from the following chapters).

SIGNING ON

The first step is to establish a telephone connection between the terminal and the computer. This procedure varies somewhat according to the installation and the equipment used. Further information about telephone numbers, types of terminals that are supported, accounting procedures, and restrictions placed on use of computer resources should be obtained from the installation. The following discussion assumes that an acoustic coupler will be used and that the terminal is capable of printing the APL symbols (a Selectric or an ASCII-APL terminal). Other terminals, such as a Model 13 Teletype, can be used (see Appendix C) but they are much less satisfactory.

1. Turn on the terminal and the coupler (sometimes one switch activates both). Dial the phone number for the computer. At most installations the phone number varies according to the type of terminal and the data rate to be used. However, at the University of Massachusetts a single phone number is used (413-545-1600). You should soon hear a high-pitched tone indicating the computer has answered the phone. Place the telephone handset in the acoustic coupler. Usually, one end of the acoustic coupler is marked "cord" to indicate which end of the telephone handset should be placed there. It is important to match the correct ends.

2. At most installations the system will begin to print after a pause of a few seconds. However, at the University of Massachusetts the user must take the initiative by typing I= and RETURN for a Selectric terminal, or J and RETURN for an ASCII-APL terminal. These first characters identify the type of terminal being used and the data transmission rate to the TEMPO mini-computer that handles communications. Failure to use the correct characters to identify the terminal is likely to result in failure to communicate.

3. The system will print something like the following:

```
75/08/14. 16.14.58.  
UMASS CYBER 74, KRONOS 2.1.7 *C*.  
USER NUMBER:
```

The first line is the current date and time, and the second line identifies the installation and version of the operating system in use. Type your user number where the system has requested it and press RETURN. The system will then request your password. Type it over the blackened squares the computer provides (or merely press RETURN if your user number has no password). The system responds with something like:

```
TERMINAL      22,COR  
RECOVER/SYSTEM.
```

4. Type APLUM to begin APL processing if you are using an ASCII-APL terminal (e.g., Teletype 38, Tektronix 4013, or Memorex 1240 with APL keyboards). At the University of Massachusetts other terminals that print APL characters (e.g., Selectric terminals) may be used without special considerations. However, for most installations the use of such terminals requires the use of APLUM commands of the form APLUM,TT-COR. See Appendix C and Appendix D for details. The APL system will identify itself as follows:

```
APLUM2.11 75/08/12. 09.57.22.  
NEW APLNEWS 75/08/06  
CLEAR WS
```

The time and date on the top line indicate when the present version of APL was generated. The message NEW APLNEWS indicates when a news item about changes in the APL system was entered. To access the news item, type the command [LOAD]*APL1 APLNEWS'. The message CLEAR WS indicates that you have begun with a clear active workspace.

IMMEDIATE EXECUTION MODE

You can now type APL expressions. What you type is evaluated immediately. For example,

```
      3+5      (You type this and press RETURN.)
8             (This is the computer's response.)
```

Pressing the RETURN key is your signal to the computer that you have finished typing the line. The computer will not process the line until you press RETURN. The expressions you type are interpreted as they appear on the paper. This is called the principle of visual fidelity. You can space forward or backward as much as you please as long as the final appearance of the paper is what you intended. If you make a typing mistake you can cancel the line by pressing ATTN and RETURN (for a Selectric terminal) or ESC (for an ASCII terminal). You can revise the line if you have not pressed RETURN. To revise the line, press ATTN (for a Selectric terminal) or LINE FEED (for an ASCII terminal) then backspace until the type element is positioned below the left-most character to be corrected. This cancels the part of the line directly above and to the right of that position. Then type any characters to replace those that were removed.

The following examples show some simple calculations being performed.

```
      2*3
8
```

```
      3÷2
1.5
```

Note that the APL system indents six spaces before allowing you to type, but the system prints its response at the left margin. This clearly distinguishes what you type from what the computer types. The following example shows how arithmetic can be performed with several numbers at the same time

```
      2*1 2 3 4
2 4 6 8
```

The series of numbers on the right is called a vector. Each element of the vector was multiplied by 2.

Values can be given a name and saved for later use. The names are called variables. The process of giving a variable a value is called assignment. The following examples show assignment of values to variables A and B.

```

A+4.8
B+1 2 3 4
A+B
5.8 6.8 7.8 8.8

```

Note that when the result of a calculation is not assigned to a variable it is printed. The sum of the elements in a vector can be found as follows:

```

B+1 2 3 4
+/B
10

```

Any symbols on the keyboard can be used as values if they are surrounded by quotes. For example,

```
GRADES+'ABADCABAABADB'
```

The = symbol can be used to compare values. The result is 1 where a match is found and 0 otherwise. For example,

```

'A'=GRADES
1 0 1 0 0 1 0 1 1 0 1 0 0

```

The following example shows how a table of comparison values can be produced:

```

'ABCD'*.=GRADES
1 0 1 0 0 1 0 1 1 0 1 0 0
0 1 0 0 0 0 1 0 0 1 0 0 1
0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 1 0

```

There is one row for each value in 'ABCD' and there is one column for each value in GRADES. To find the number of A's, B's, C's, and D's, add up the 1's in the four rows as follows:

```

+/'ABCD'*.=GRADES)
6 4 1 2

```

Below is an example of another comparison table using < instead of =. Also, instead of using +/ to add the rows as in the last example, +/ is used to add up the columns. The symbol /, called an overstrike, is formed by typing /, backspacing, and then typing -. (Actually the two symbols comprising the overstrike can be typed in either order.)

```

V+2.1 3.2 .08 8.1 4.6 1.2 2.3 4.2 1.6
2 4 6 8*.<V
1 1 0 1 1 0 1 1 0
0 0 0 1 1 0 0 1 0
0 0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0 0

```

```

      +/(2 4 6 8<=V)
1 1 0 4 2 0 1 2 0

```

As shown, this operation classifies each value in *V* according to the number of values in 2 4 6 8 it exceeded. That is, a value between 2 and 4 is in class 1, a value between 4 and 6 is in class 2, and a value between 6 and 8 is in class 3. A user defined function can be written to perform this operation:

```

VZ←A CLASSIFY B
[1] Z←+A*.1B V

```

The first *V* signals to the computer that you wish to define a function. The first line shows that the function takes two arguments (input values) and gives a result. The computer numbered the next line with [1]. The *V* at the end indicates you have completed typing the lines of the function. The function can be used as follows:

```

      2 4 6 8 CLASSIFY V
1 1 0 4 2 0 1 2 0

```

Now it might be interesting to tabulate how many 0's, 1's, 2's, and so forth were in the last result. An APL function can be written to do this but it requires two more APL operations: The largest value in a vector *V* is given by *F/V*; and *iN* gives the integers 1 2 3 ... *N*. We use both of these as follows:

```

VZ←TAB B
[1] K←1+((F/B)+1)
[2] K←K 1
[3] Z←+/(K-. B)V

TAB 0 1 0 1 2 1 3
2 3 1 1

TAB(2 4 6 8 CLASSIFY V)
3 3 2 0 1

```

The following function will give a crude histogram of these results:

```

VZ←HIST B
[1] P←F/B
[2] Z←((P+1)-1P)*.5B V

HIST 3 3 2 0 1
1 1 0 3 0
1 1 1 0 0
1 1 1 0 1

```

A neater histogram can be produced by adding another line to the *HIST* function:

```

      )HIST                                (Function definition is opened.)
[3]  Z←' '[Z+1]                            (Another line is added.)
[4]  []                                     (Display is requested.)
      V Z←HIST B
[1]  P←1/B
[2]  Z←((P+1)-1P)°.5B
[3]  Z←' '[Z+1]
      V                                     (Definition is closed.)
[4]  V

```

Note that to add more to the function you first type *v* and the name (but not *Z←HIST B*). The computer numbered the line [3]. Typing [] on the next line caused the computer to list the function. Finally, the *v* was typed to indicate that no more lines were to be added. This function can now be used with the two others as follows:

```

      HIST TAB 2 4 8 8 CLASSIFY 7
[]
[]
[]

```

You can display the names of defined functions and variables as shown below:

```

      )FNS
CLASSIFY HIST TAB

      )VARS
A B GRADES K P V

```

To save the functions and variables for use at some other session, type

```

      )SAVE MYWORK

```

The collection of functions and variables constitutes a workspace. Here a workspace named *MYWORK* was saved. It is advisable to save the workspace often if you are changing it in order to minimize the amount of work that will be lost in the event of a serious computer malfunction. (See Chapter 12 for the procedure to follow to avoid losing work after a telephone disconnect or minor computer malfunction) To remove all functions and variables from the workspace you are now working with, type

```

)CLEAP
NEW APLNEWS 75/08/06
CLEAP WS
)FNS
      (No functions.)
)VARS
      (No variables.)

```

You can retrieve the MYWORK workspace as shown below:

```

)LOAD MYWORK
MYWORK 75/08/06 16:18:28
)FNS
CLASSIFY HIST TAB

```

To terminate the session and sign off the computer, type)OFF.

```

)OFF
A123456 LOG OFF. 18.12.07
A123456 CP 0.017 SEC

```

Although this sample session was short and only used a small fraction of the APL operations, it illustrates how well adapted APL is to experimentation. Programs can easily be developed in small parts and put together to do useful work. The flexibility in using functions in new combinations makes many problems much easier to solve. Many users of APL begin with the habit (formed by familiarity with other computer languages) of writing large monolithic programs in one piece. It should be evident that the modular approach illustrated above is better.

Chapter 1. User-Defined Functions

Function definition mode allows the user to enter function lines one at a time, remove lines, change lines, insert lines, or display the function. In function definition mode, APL statements entered are not executed or checked for errors, nor are system commands executed. Most errors will be detected when the statement is executed for the first time. System commands are illegal in the body of a function.

CREATING A FUNCTION

To enter function definition mode, type `⍝` and the function header. The form for the function header should be determined by how the function is used. The six possible forms are shown in the following table.

Number of Arguments	0	1	2
No Result	<code>name</code>	<code>name B</code>	<code>A name B</code>
Result	<code>Z+name</code>	<code>Z+name B</code>	<code>Z+A name B</code>

The name of the function (represented by `name` in the table above) can consist of any number of letters `A` to `Z`, underscored letters `A` to `Z`, digits `0` to `9`, or the symbols `_`, `Δ`, or `Δ`, but must not begin with a digit. The function name must not be in use for another global function or global variable. In the table, `Z` is used as the result variable, `A` is the left argument, and `B` is the right argument. Any other names could be used instead, provided they are used consistently in the body of the function. Names of system functions or variables must not be used as the result variable or argument variables.

After any of the forms in the table, there can be a semicolon and additional names separated by semicolons. The additional names declare variables and functions to be local to

Summary of Function Definition.

Creating a Function

VZ+A NAME B

Reopening Definition

VHANZ

Display

[U] (Display all.)
[20] (Display from 20.)
[20] (Display line 20.)

Insert a line between [2] and [3]

[2.1] P+15

Delete line [3]

[Δ3]

Replace line [3]

[3] P+Q+5*1#

Line Editing for line [3]

[3] [3]8]	(Line 3, column 8.)
[3] P+Q+5*1#	(The line is printed.)
/1 1	(Type / to remove, 1 to insert 1 space.)
[3] P+ +5*1 #	(Type additions in the spaces.)

Extending line [5]

[500]

Context editing for line [3]

[3] /.old phrase.new phrase	(To replace.)
[3] /.old phrase.	(To delete.)
[3] /..new text	(To extend.)
[3] /..	(To display the line and then extend it.)

the function. (Local variables and functions are discussed later in this chapter.)

The function header is line [0] of the function. After entering a `v` and a header, function definition is said to be open. The system then types [1] on the next line to invite the user to enter line [1] of the function. The user can then type function lines, and the system continues to number lines. When the last line has been entered, function definition mode can be terminated by typing a `v` at the end of a line or on a line by itself. The `v` is recognized as long as it is the last nonblank character on the line, even if the line is a comment.

Upon an attempt to close definition, the function header is checked for duplicate use of names, and statement labels are checked for duplication with names used in the header or names used for labels on other statements. In addition, the form of the header is checked for correctness. Any of these errors causes the message `DEFN ERROR` and display of the header or the line with the incorrect label. The error should be corrected, then `v` should be typed to attempt to close definition again.

REOPENING DEFINITION

To add more lines to a function, first reopen definition by typing `v` and the name. No other header information should be used--use of other header information causes the system to assume you are mistakenly attempting to create a new function having the same name as an old function. (The header can be changed after definition is open by treating it as line [0] and revising it as described below.) After definition of the function has been opened, the system types the number the next line will have. The user can type additional lines in the same manner as when the function was created.

OVERRIDING THE LINE NUMBER

After the system types a line number, the user can override that line number by providing a different one. For example, assume the system printed [4] because line [4] was expected. The user could type [2] to override the [4] if he wants to enter a new line [2]. He could type the new line [2] on the same line he types the line number, or, he can type only the overriding line number and press RETURN, after which the system would type [2]. After line [2] is provided, the system would continue by numbering the next line with [3].

To insert a new line between lines, use a fractional line number. For example, [3.2] could be used to insert a line between lines 3 and 4. No more than 4 digits are allowed after the decimal point. The system continues to number subsequent lines by incrementing the last position of the overriding line

number until another overriding line number is used. Thus, after [3.98] would follow [3.99], [4], [4.01], and so forth.

To remove a line, use a request of the form [a3]. The tilde before the overriding line number indicates that the line should be deleted. More than one line number can be provided (e.g., [a3 9 1.6]). Note that a line cannot be replaced by a blank line by overriding a line number with the number of the line to be deleted and pressing RETURN.

Line [6] (the header) can be replaced like any other line, but it cannot be deleted. If the new line [3] causes the name of the function to change, the old function remains as it was when function definition was opened, and a function having the new name is produced when definition is closed. The function name cannot be changed to the name of a global function or variable.

When function definition is closed, all lines are renumbered with consecutive integers. Because line numbers can change, use of labels for all branching is recommended.

If a *WS FULL* error occurs during function definition, function definition mode is closed automatically, and all functions and variables remain as they were when function definition was opened.

DISPLAY OF FUNCTIONS

When function definition mode is open, the entire function can be displayed by typing [0]. To display only line 3 of the function, type [30]. To display all lines from line [3] on, type [03]. If you interrupt the display (see Appendix C), function definition remains open unless a closing *?* appeared in the same line as the request for display.

LINE EDITING

Line editing can be used to change individual characters in a line. To begin line editing, type something of the form [308], where 3 is the number of the line to be revised, and 8 is the approximate position in the line where the first change is to be made. The system then prints the line and unlocks the keyboard below the 8th character. Use spaces or backspaces to position the typeball to the position to be changed. Type / under a character to delete it, or type a digit 1 to 9 to insert 1 to 9 spaces before the character, or type A below it to insert 1 space, B for 10 spaces, C for 15 spaces, and so on up to H for 40 spaces. To replace a character, you must delete that character (which closes up the line leaving no new space) and type a 1 before the next character to provide space for the replacement character. After the changes are specified and

RETURN is pressed, the system prints the revised line and waits at the position of the first inserted space or at the end of the line if no spaces were inserted. Type in any new characters in the spaces and then press RETURN.

If line editing causes the line number to change, the old line remains intact, and a new line with the new number is inserted. To extend a line, use the form [3[0]. The zero as a position in the line causes the line to be printed and the keyboard to unlock at the end of it.

Note that line editing should not be used with a terminal that does not print the APL character set. The APL system does not take account of single APL characters printing as more than one character on such terminals, so the position where changes are made can be somewhat unpredictable.

CONTEXT EDITING

Context editing allows replacement of the first occurrence of a given phrase by another phrase. Context editing is often more convenient than line editing when the changes are localized in a small part of the line and prior display of the line is not required. The line that is changed is the line the system is expecting next. The editing command has the form

/old phrase.new phrase

The */* signals that what follows is an editing request. The symbol immediately after the */* is the symbol chosen by the user to mark the end of the first phrase and the beginning of the second phrase. Any symbol can be used as long as it occurs in neither phrase. The delimiter can optionally be used at the end of *new phrase*, but if it is not, the *new phrase* is assumed to end at the rightmost excursion of the type element (except that a *?* at the end is not considered part of the phrase). The system first searches for the *old phrase*. If the *old phrase* is found, it is deleted and the *new phrase* is inserted at that point. If the *old phrase* is empty, the *new phrase* is inserted at the end of the line. After replacement, the system prints the corrected line. If both the *old phrase* and *new phrase* are empty, the system prints the line and unlocks the keyboard to allow the line to be extended. If the *old phrase* is not found, the error message

13: PHRASE NOT FOUND

is printed. (Note that the line number can be changed using context editing. When the line number is changed, the old line remains intact and the revised line is added to the function.) If the editing request is incorrectly formed, *DEFN ERROR* is printed. The following examples illustrate useful editing requests:

<code>1.FOUR.SIX</code>	(FOUR is replaced by SIX)
<code>1.3.5.4.5</code>	(A comma is used as the delimiter because periods occur in the phrase.)
<code>[3] 1.X\Y+.</code>	(Deletion of X\Y+; [3] was used to override the line number that had been printed by the system.)
<code>1...C</code>	(To extend the line with :C)
<code>1..</code>	(To extend the line with information from the keyboard.)

FUNCTION DEFINITION SHORTCUTS

In general, a line you type in function definition mode is used up before you are required to type another line. For example, you can type `[03]V` to display line [3] and then close function definition. Or, you can type `VFN[3]P+1N7` to open definition, override the line number with [3], provide a new line [3], and close definition. A `V` at the end of a statement is always recognized, but other editing requests at the end are interpreted as being part of the line. Hence `VFN[3]P+1N[4]V` would cause line [3] to be `P+1N[4]`. It would not cause display of line [4] after replacing line [3].

LINE SEPARATOR

You can use the diamond symbol (the overstrike `*` for a Selectric terminal) as an input line separator for function definition mode. The parts separated by diamonds are used as if they were entered consecutively from the keyboard except that the normal line number prompt is suppressed. However, input lines for line editing requests must still be entered separately from the keyboard. Any diamonds preceded by an odd number of quotes are considered to be part of character constants and not line separators. If an error occurs, any remaining lines are discarded and input is again requested from the keyboard. The following example shows use of the line separator to define a function and then display it:

```

VZ+NEXTLINE N * Z+CFREAD N * Z+(V\Z*' ')/Z * [ ]V
VZ+NEXTLINE N
[1] Z+CFREAD N
[2] Z+(V\Z*' ')/Z
V

```

The purpose of the line separator is to reduce waiting time when the computer responds slowly. The diamond is allowed as a line separator only in function definition mode and should not be confused with the use of the same symbol in other versions of APL to allow multiple executable APL statements on a line.

LOCALIZATION OF VARIABLES AND FUNCTIONS

The variables local to a function include all variables appearing in the function header and all statement labels. Variables that are not local to any function are called global variables. When execution of a function begins, the local variables take precedence over any other functions and variables having the same names. Other variables that were in effect before this function was called (that is, those not local to this function, which are called variables global to the function) remain accessible. When execution of the function is completed, the variables local to it vanish, thus releasing storage space for other uses, and any variables or functions global to the function become accessible again.

As execution of the function begins, the argument variables are assigned the values of the arguments in the expression invoking the function. If the function modifies the arguments, it is actually changing a copy of the original arguments. (See Chapter 11 for storage implications.) The label variables are also assigned scalar integer values of the line numbers on which they appear. These variables are locked to prevent them from being assigned inappropriate values. (However, they can be given improper values if they are first erased and then given a value.) The result variable and any other variables listed after the first semicolon in the header have no initial value.

A function can also have another function local to it if it has the second function's name in its header. As for local variables, the local function is undefined as execution of the main function begins. The local function can then be defined by use of `DEF` or `COPY` with `ENV` having 1 as its value (the normal case -- see Chapter 7 for details about `DEF`, `COPY`, and `ENV`). When execution of the main function completes, the function local to it will vanish, just as a local variable would, and any temporarily inaccessible function or variable having the same name would again become accessible.

FUNCTION EXECUTION

Function execution begins when the name of the function is encountered in an expression being executed and any arguments have been evaluated. The system must save information about how far execution has progressed in the calling line in order to be able to eventually return to it and continue processing. The state indicator is a summary of this information and is available to the user. Execution of a function begins with establishment of local variables as discussed in the last section. Then, except for branching, the statements are executed in order from first to last. After the last statement has been executed, the value last assigned to the result variable is returned to be used in the calling expression, and all local variables vanish.

Branching can be used to control which statement will be executed next. A branch statement consists of a branch arrow followed by an expression that returns a result. The value must be a scalar or a vector, and unless it is an empty vector, the first value must be a nonnegative integer. If an empty vector is used, the next statement is performed. If the value is a scalar or vector, its first element is used as the number of the line to be executed next. If the value is 0 or exceeds the largest line number, the function exits. The following examples show useful branch statements. Close examination of the expressions to the right of the arrows should show how they generate appropriate line numbers:

```
+5*1A<14      (Branch to line 5 if A is less than 14. Note
               that this will not work in 0-origin.)

+(A=3)/8       (Branch if A equals 3 to line 8.)

+(L1,L2,L3)[2*+5]
               (Branch to L1 if B is negative, to L2 if
               zero, or to L3 if positive.)

+(A>20 18 13 2)/L5,L4,L3,L2
               (Branch to L5 if A is greater than 20, branch
               to L4 if greater than 18 but not 20, to line
               L3 if greater than 13 but not 18, to L2 if
               greater than 2 but not 13, or go to the next
               line if A is less than or equal to 2.)
```

STATE INDICATOR

Any lines that call for execution of another function cannot be completed until the other function has exited. Such unfinished lines are called pendent lines. If an error causes a halt at a line of a function, that halted line is said to be suspended. The state indicator is a record of all pendent and suspended lines of functions. It omits partially executed lines entered in immediate execution mode, lines entered for quad input, and lines used as arguments to the execute function. The state indicator with variables, displayed by the system command)SIV, shows what lines are pendent or suspended and also shows variables local to functions. An abbreviated form, displayed by the system command)SI, omits names of label variables and names appearing in the header after the first semicolon. For example:

```
)SIV
[3]=2*PRINT B;X;K:LIMIT:L1:L2
[4] SIMU K:L3
```

```

)SI
[3.*Z-PRINT B
[~] SIMU X

```

In both examples above, the most recently invoked line is shown first. An asterisk marks a line that is suspended. Here, line [4] of *SIMU* called *PRINT*, and execution of *PRINT* halted at line [3] because of an error. The *)SIV* display shows the full function header followed by a colon and names of statement labels separated by colons. If the function has no statement labels, no colons appear.

The *)SIV* display shows that the variable *X* currently accessible is the one local to *PRINT*. The other *X* local to *SIMU* is no longer accessible. However, the label variable *L3* local to *SIMU* still has its value because no variable *L3* is local to *PRINT*. In general, the current value associated with a variable name is that for its first occurrence on the state indicator. If it does not appear on the state indicator, the current value is that of any global variable having that name.

A branch in immediate execution mode can be used to restart execution of the most recent suspended function. For example, +5 would cause execution of *PRINT* to continue at line 5. Usually, the function would be corrected or values of variables would be changed before proceeding. To remove the most recent suspension and the pendent lines that led to it, type a branch arrow with nothing to the right. A beginning user of APL often begins a new execution of a function without removing the old one, causing a large number of suspensions to accumulate. These unnecessary suspensions waste space and can lead to confusion by allowing local variables to make global variables inaccessible. When a suspension occurs, it is a good practice to either make corrections and continue execution or clear the state indicator by use of the niladic branch (see Chapter 2). An excessive number of suspensions can be eliminated by use of 0 [SAVE 'name'] (see Chapter 7).

The information the system keeps about pendent lines can become invalid if the pendent functions are altered, replaced, or erased. The system responds by printing 14: *SI DAMAGE* and surrounding with brackets the names of the affected functions on the state indicator display. Execution of the affected functions cannot be resumed. Experienced users are expected to avoid *SI DAMAGE* if they intend to continue execution of a halted function. Certain changes to suspended functions can also lead to *SI DAMAGE* --specifically, altering the function header or changing the number or relative order of statement labels.

HALTING A FUNCTION

While a function is running, it can be halted by an interrupt (see App. C). However, when the keyboard is unlocked,

use of the interrupt on some terminals is interpreted as an attempt to revise the line being entered. To halt a function requesting quote-quad input, type the overstrike # (formed from O, U, and T). This results in suspension as if an error had occurred. To halt a function requesting quad input and remove it and all related pendent lines from the state indicator, use a branch arrow with nothing to the right.

TRACE AND STOP CONTROLS

Any stop, trace, and timing controls in effect for a function are cleared if function definition mode is used to change the function in any way.

LOCKED FUNCTIONS

A function can be locked by using $\#$ (V overstruck by ~) in place of V when opening or closing function definition. Locking a function prevents display of the function and prevents its definition from being reopened. An attempt to open definition of a locked function results in the error message *DEFN ERROR*. A locked function cannot be unlocked; if you will want to change a locked function at a later date, keep an unlocked copy of the function in another workspace protected by a password or keep a printed listing of the function. If closing definition of the function results in *SI DAMAGE*, the request to lock the function is ignored.

Chapter 2. Statement Form and Order of Evaluation

This chapter discusses the form of legal APL statements and the order of evaluation of statements. Restricting the discussion to "APL statements" means that system commands (which are distinguished by beginning with a right parenthesis) are not of interest here. The meaning of a statement is determined in part by its form, but mainly by the functions used and the environment in which they are used. This chapter discusses the influence of form on meaning and leaves the functions and environment to be discussed in several other chapters.

NOTES

The use of spaces in an APL statement is usually unimportant to the meaning of the statement except for a few cases:

(1) Names must be separated from other names by spaces, and names must be separated from digits of a number to the right by spaces. (Also, a name beginning with *F* must be separated from digits to the left.) Otherwise, they would run together and appear to be all one name. Conversely, spaces in the middle of a name would make it appear to be two names.

(2) Numbers next to one another must be separated by spaces, and spaces cannot appear within a number.

(3) Spaces within a character constant are treated as any other character in the constant and affect the value of the constant.

(4) Spaces in a comment (except for trailing spaces) are preserved by the system. Although they have no meaning to the APL system, they may be important to the reader of the comment.

FUNCTION DEFINITION AND SYSTEM COMMANDS

As execution begins for statements entered in immediate execution mode, entered in response to quad input, or used as arguments to the execute function (but excluding statements in the body of a function), a check is made to determine if the first nonblank character on the line is `V`, `Y`, or `.`. In these cases the statement is preconverted to become a call to the function `DFD` (a program that performs function definition mode) or `SY` (a program that performs system commands) with the original line as a character argument. For example, `VFN[6]` becomes `DFD 'VFN[6]'`. To preserve the original meaning, any quotes in the original statement become double quotes after the conversion. Any comment at the end of the original statement becomes part of the argument to `DFD` or `SY`. The discussion that follows assumes that any such preconversion has already been performed.

COMMENTS

A comment may be entered in immediate execution mode or may appear in a function line. Comments begin with the symbol `*` and extend to the right to the last nonblank on the line. The part of the line following the comment symbol is not executed. This allows the user to intersperse descriptive text with APL statements. The following example shows a comment used in immediate execution mode to add a description to the printed transcript of the session:

```
X←2×1/N A TO GENERATE 2 4 6, ETC.
```

The following discussion makes no further mention of comments, although a comment may appear at the end of any line, or the comment may constitute the entire line.

CONSTANTS

Constants represent numbers or characters. For example, `.14 5.2 9` is a numeric constant-vector, and `'ABCD'` is a character constant-vector. Constants consisting of one character or number are scalars, while those having more components or no components are vectors.

An *unsigned-number* is defined to be any of the following:

```
digits  
digits.digits  
.digits
```

where *digits* represents one or more of the digits 0123456789. The italic notation used here is used throughout this book to

denote a term having a special definition. Here, *digits* represents a sequence of digits, not the letters *d i g i t* and *s*. Hence the following numbers are examples of *unsigned-numbers*:

3.4
.05
58

However, an *unsigned-number* cannot end with a decimal point. Hence, 3., would not be legal.

A number has any of the following forms:

unsigned-number
~unsigned-number
unsigned-number exponent
~unsigned number exponent

The symbol *~* is used to express a negative number--the minus symbol cannot be used in its place. An exponent has one of the following forms:

*E**digits*
*E**digits*

The *E* can be read "times 10 to the power." So, *:E2:* means $1 \times 10 \times 23$, and 3.25×3 is the same as .0032. A *numeric-constant* is formed from one or more number, separated by spaces.

A *character-constant* is of the form:

'symbols'

where *symbols* represents any number of APL symbols, including no symbols. The symbol *'* in a *character-constant* is represented by two quotes. For example,

'IT'S'
IT'S

Quotes must always appear in pairs. An expression with an odd number of quotes results in a *SYNTAX ERROR*.

The term *constant* means either a *numeric-constant* or a *character-constant*.

FUNCTIONS

Functions are of three kinds:

(1) System functions, which have names that begin with *]* or *⌈*, are used to communicate with the APL system.

(2) User-defined functions, which have names formed in the same way as variable names, are the only ones the user can define.

(3) Primitive functions (except those produced by operators) are symbolized by single characters such as +, *, /, etc.

For the purposes of this chapter, the important features of functions are the number of arguments they require and whether they return results. Functions can be monadic (one argument), dyadic (two arguments), or niladic (no arguments). If *rfunction* is used to denote a function that returns a result and *function* is used to denote one that does not, the six possible forms are:

<i>dyadic-rfunction</i>	(Dyadic, returns a result.)
<i>monadic-rfunction</i>	(Monadic, returns a result.)
<i>niladic-rfunction</i>	(Niladic, returns a result.)
<i>dyadic-function</i>	(Dyadic, no result.)
<i>monadic-function</i>	(Monadic, no result.)
<i>niladic-function</i>	(Niladic, no result.)

For some primitive functions and system functions the same symbol or name is used for two distinct functions--one monadic and the other dyadic. The dyadic function is used if there is a left argument, and the monadic function is used if there is no left argument.

Dyadic user-defined functions can be used without a left argument, but if the function requires a value for its left argument, a *VALUE ERROR* results. The following example is a function that can be used without a left argument provided its right argument is not negative:

```

      VZ+A P1 B
[1] Z+2*B
[2] +(B>0)/0
[3] Z+Z+A V

      P1 5
10

      P1 ^1
05: VALUE ERROR
P1[3] Z+Z+A
/

      5 P1 ^1
3

```

The function *QNC*, described in Chapter 7, can be used to check whether the left argument has a value. This could be used to write user-defined functions that have distinct monadic and

dyadic forms in analogy to distinct primitive functions having the same symbol.

Whether a name refers to a function or a variable is a matter that can be decided only when the line begins to execute. Also, whether a function actually returns a result may depend on circumstances. For example, if a user-defined function was defined to return a result, but the result variable was not assigned a value prior to exit from the function, a *VALUE ERROR* results if the expression calling the function requires a result.

OPERATORS

An operator is a special kind of function that takes functions as arguments and produces functions as results. Following are examples of four types:

```
A+.B (Inner Product.)
A*.B (Outer Product.)
*/B (Reduction.)
*\B (Scan.)
```

The operators are the period, /, and \. In place of the + and * in the above examples, any dyadic scalar function symbols could be used. These operators are discussed in detail in Chapter 6, but for the present, it is important to note that the forms exemplified by +.* and *.* represent dyadic functions that return results, and */ and *\ represent monadic functions that return results.

The axis operator is used to specify the coordinate along which an operation is to be performed. Only a few functions can be used with the axis operator and further details are discussed with those functions. The operator is used in the form *function-symbol*[*value*]. For example:

```
⊕[2]B
*/[1]B
```

VARIABLES

A variable is a name that might be associated with a value. The variable-name is formed from any sequence of the letters A to Z, underscored letters A to Z, digits 0 to 9, or the symbols \$, @, or _, but the name cannot begin with a digit. System variables are special variables with names that begin with \$ or @. The rest of the name can be composed in the same way as normal variable names. Only the system variables recognized by the system can be used--the user cannot invent new ones.

An indexed-variable is of the form:

variable[list]

A variable-name having no value associated with it can be used only immediately to the left of an assignment arrow; otherwise a VALUE ERROR will result.

VALUES

A value is any of the following:

variable
constant
indexed-variable
monadic-rfunction value
left-argument dyadic-rfunction value
niladic-rfunction
left-argument
variable-name+value
indexed-variable+value
(value)
+value

The last case has the further restriction that the + may appear only as the first character of a line.

Use of an indexed-variable to the left of a specification arrow sets the values of elements of the variable without changing the shape of the variable. Used elsewhere, the index returns parts of a value.

The assignment arrow can be used to give a value to a variable or to change the value of a variable. The result of the assignment (not to be confused with the value of the variable) is the value used on the right. Consequently, $A+B[1\ 2]+3$ is the same as the two statements $B[1\ 2]+3$ and $A+3$. Similarly, $A+[]+B$ is the same as $[]+B$ and $A+B$; but $A+[]+B$ is not the same as $[]+B$ and $A+[]$.

The operations to find a value occur in right to left order. Hence, $3*2+4$ means $3*(2+4)$. When a dyadic function is encountered, the right argument is preserved while the expression producing the left argument is evaluated. Hence,

$A+3$
 $(A+4)*A$

12

However, it is poor programming practice to take advantage of the right argument being preserved; some APL systems produce a different result in cases like the one above. More generally,

any value encountered in the right to left scan is preserved.
For example,

```
4 4 5 5
A[3 2 1]←A
A
```

6 5 4

(On some APL systems the result would be 4 5 4 or 6 5 6 because the variable on the right is not preserved, while on other systems such operations are prohibited.) However, the following example shows a case where the value is not preserved because the scan has not reached the variable:

```
A←2
A+A←3
```

6

LEFT ARGUMENTS

A left-argument is any of the following:

```
variable
constant
constant[list]
indexed-variable
(value)
(value)[list]
niladic-rfunction
```

For example, γ can be used as a left argument, *ALPHA* can be used as a left argument, and '123'[2] can be used as a left argument, but 2×3 cannot unless it is enclosed in parentheses. In fact, in $2 \times 3 \times 5$, the 3 would actually be used as the left argument to \times .

EXPRESSIONS

An expression is the same as a value except that it need not return a result that can be used for subsequent operations. An expression is any of the following:

```
monadic-function value
left-argument dyadic-function value
niladic-function
value
→
```

The last case, called niladic branch, can be used only as the leftmost character of a line. The branch with no value or expression to the right causes exit from the executing function and from all other functions on the state indicator up to any

previous suspension.

LISTS

A list is of the form:

```
list-element  
list element;list-element  
list-element;list-element;list element ...
```

The list, if used for an index, must have one list element for each dimension of the array being indexed.

A list-element can be:

```
vacant  
value  
expression
```

An expression that does not give a result can be used in a list used for indexing and is treated as if the list element were vacant. A list element is vacant if there is nothing at all in that position. For example, $P[3;]$ illustrates a list having a vacant list-element.

The elements of a list are evaluated in right to left order. Hence

```
A-3 ; A+5
```

gives A a final value of 3. Note that the semicolon is not an APL function. Lists can only be used for indexing and output. Expressions like the following are illegal:

```
3p(A;B)
```

Also, the statement

```
3pA;B
```

is equivalent to

```
(3pA),B
```

not

```
3p(A;B)
```

The expressions separated by semicolons are evaluated separately, then their results constitute the list.

LINES AND IMPLICIT OUTPUT

A line is any of the following:

value
expression
list
vacant

When a line is a list, the list elements are printed in left to right order. The list can contain a mixture of character and numeric values as shown below:

```
X+3=
'THE VALUE OF X IS- ' ; X
THE VALUE OF X IS: 34
```

Scalar and vector list elements are printed on the same line (if `QPW` has not been exceeded), but printing of a matrix or array of higher rank begins on a new line, and any subsequent vector or scalar begins on a new line. List elements that are vacant or that produce no results are skipped over.

If the first list element is a niladic branch, no output is produced. If the first list element is a branch with a value to the right, the value of the branch is printed along with the other list elements, then the branch is taken.

When the line is a value, the value is printed unless a specification or branch occurred as the last operation. Hence, `3+2` would print a result, but `A+3+2` or even `(A+3+2)` would not.

STATEMENTS

A statement is either a line or a line with a label. The label is a variable-name and colon placed before the line. For example:

```
REPEAT:→4×1X=Y
```

A label on a statement entered in immediate execution mode, for quad input, or in the argument to the execute function is ignored.

QUAD AND QUOTE-QUAD

The system variables `Q` and `Q` are used for input and output. When they are assigned a value, the system prints the value. When their values are used in an expression, the system reads input from the keyboard to provide the value.

When `Q` input is requested, the keyboard unlocks (normally

with the type element at the left margin). Any characters typed are returned as a vector, except that a single character gives a scalar.

When `␣` input is requested, the system prints `␣:` and then on the next line indents six spaces and unlocks the keyboard. Any APL expression that returns a result can be entered. If the expression is incorrect or does not produce a result, an error message is printed and the input request is repeated. For example,

```

      A←␣
␣:      B      (This is the input line.)
05: VALUE ERROR
      B
      /
␣:      (The input request is repeated.)
      2×14
      ||
2 4 6 8

```

A branch in quad input does not actually effect a branch.

The `␣` can be used for output to conserve lines in a program. The statement `␣←A+B` has the same effect as the two statements `A+B` and `␣←B`.

The `␣` symbol, when used for output, is slightly different from `␣` used for output. Ordinarily, APL output is followed by a carriage return so that the next input or output will begin on a new line. However, when quote-quad is used for output, the extra carriage return is suppressed. This allows the program to continue output on the same line or to give output and then request input on the same line. For example,

```

      VZ←ASK B
[1] ␣←B
[2] Z←␣ V

      P←ASK 'AGE? '
AGE? 38
      P
      38      (Note leading blanks in the result.)

```

The leading blanks show where the typeball was positioned when the keyboard was unlocked. The person who was typing could have backspaced and replaced the blanks with other characters. Any leading blanks can be removed by using `(v\B=' ')/B←␣`. Note that `␣PW` is not ignored when `␣` output is used. If the number of printed characters reaches `␣PW`, the system inserts a carriage return in the output and indents 6 spaces before continuing the output.

Chapter 3. Scalar Functions

The class of scalar functions includes those functions that can be defined for scalar arguments and then can be extended to other arguments through element-by-element extension. That is, if the function is monadic, the result has the same dimensions as the argument, and the elements of the result are found by applying the function to all elements of the argument. For the dyadic functions the following rules apply:

1. If the arguments have the same shape, the result has that shape and is formed by applying the function to the corresponding elements of the arguments.
2. If one argument is a one-element array and the other is not, the result has the shape of the one that is not one element. The one-element argument is used with each element of the other argument to form the result.
3. If both arguments are one-element arrays, the result has the larger of the ranks of the arguments.

For the dyadic functions, the arguments must either have identical shapes or at least one must be a one-element array. Any other arguments produce a *RANK ERROR* if their ranks differ, or a *LENGTH ERROR* if their ranks match but dimensions differ. The following examples illustrate some of these rules:

[+4*3 3p19]

1	2	3
4	5	6
7	8	9

(A monadic scalar function.)

-1	-2	-3
-4	-5	-6
-7	-8	-9

Summary of Scalar Functions.

Dyadic Function		Monadic Function	
$A+B$ Addition	Sum of A and B . $3+5 \rightarrow 8$	$+B$ Plus	Same as $0+B$.
$A-B$ Subtraction	A minus B . $3-4 \rightarrow -1$	B Additive Inverse	Same as $0-B$.
$A \times B$ Times	Product of A and B . $2 \times 4 \rightarrow 8$	$\times B$ Signum	Sign of B . Same as $(B > 0) - B < 0$ $\times 3 \ 0 \ -2 \rightarrow 1 \ 0 \ -1$
$A \div B$ Divide	A divided by B . Division by 0 is not allowed except that $0 \div 0$ is defined to be 1. $3 \div 2 \rightarrow 1.5$	$\div B$ Reciprocal	Same as $1 \div B$. Not allowed if B is 0. $\div .2 \rightarrow .5$
$A \uparrow B$ Maximum	Larger of A and B . $3 \uparrow 5 \rightarrow 5$ $-1 \uparrow -5 \rightarrow -1$	$\uparrow B$ Ceiling	If B is an integer, the result is that integer. Otherwise the smallest integer greater than B . $\uparrow 2.5 \ 3 \rightarrow 3$
$A \downarrow B$ Minimum	Smaller of A and B . $3 \downarrow 5 \rightarrow 3$ $-1 \downarrow -5 \rightarrow -5$	$\downarrow B$ Floor	If B is an integer, the result is that integer. Otherwise the largest integer less than B . $\downarrow 2.5 \ 3 \rightarrow 2$
$A \wedge B$ Power	A to the B power. A may be zero if B is not negative. $0 \wedge 0$ is defined to be 1. If $A < 0$, B must be representable as a rational fraction with an odd denominator.	$\wedge B$ Exponential	e to the B power (e is 2.718281828...)
$A \# B$ Logarithm	Base A logarithm of B . A must be positive and must not be 1.	$\# B$ Natural Logarithm	Natural (base e) logarithm of B .

Summary of Scalar Functions, Continued.

Summary of Several Functions, Symbols

$A \ B$ Residue	The remainder of B divided by A . More precisely, $B \ A \ B \div A \ A \rightarrow 0$	B Magnitude	Absolute Value of B . $13 \ 0 \ ^{-3} \rightarrow 3 \ 0 \ 3$																														
$A!B$ Combinations of	Number of combinations of B things taken A at a time for positive integer arguments. More generally $A!B \rightarrow ((!B) \div (!A)) \times !B - A$	$!B$ Factorial	Factorial of B for nonnegative integers. Otherwise the Mathematical gamma function of $B+1$. Not defined for negative integers.																														
$A \circ B$ Circular	The argument A determines which function from the following table is applied to B . A must be an integer in the range -7 to 7	$?B$ Roll	A random choice from B . Depends on current origin.																														
		$\sim B$ NOT	B must consist of 1's or 0's. $\sim 1 \rightarrow 0 \quad \sim 0 \rightarrow 1$																														
		$\circ B$ Pi times	Pi times B $01 \rightarrow 3.14159...$																														
<table> <tr> <th>N</th><th>$N \circ B$</th><th>$(-N) \circ B$</th></tr> <tr> <td>0</td><td>$(1-B \times 2) \times .5$</td><td>$(1-B \times 2) \times .5$</td></tr> <tr> <td>1</td><td>$\sin B$</td><td>$\arcsin B$</td></tr> <tr> <td>2</td><td>$\cos B$</td><td>$\arccos B$</td></tr> <tr> <td>3</td><td>$\tan B$</td><td>$\arctan B$</td></tr> <tr> <td>4</td><td>$(1+B \times 2) \times .5$</td><td>$(1+B \times 2) \times .5$</td></tr> <tr> <td>5</td><td>$\sinh B$</td><td>$\operatorname{arcsinh} B$</td></tr> <tr> <td>6</td><td>$\cosh B$</td><td>$\operatorname{arcosh} B$</td></tr> <tr> <td>7</td><td>$\tanh B$</td><td>$\operatorname{artanh} B$</td></tr> </table>			N	$N \circ B$	$(-N) \circ B$	0	$(1-B \times 2) \times .5$	$(1-B \times 2) \times .5$	1	$\sin B$	$\arcsin B$	2	$\cos B$	$\arccos B$	3	$\tan B$	$\arctan B$	4	$(1+B \times 2) \times .5$	$(1+B \times 2) \times .5$	5	$\sinh B$	$\operatorname{arcsinh} B$	6	$\cosh B$	$\operatorname{arcosh} B$	7	$\tanh B$	$\operatorname{artanh} B$				
N	$N \circ B$	$(-N) \circ B$																															
0	$(1-B \times 2) \times .5$	$(1-B \times 2) \times .5$																															
1	$\sin B$	$\arcsin B$																															
2	$\cos B$	$\arccos B$																															
3	$\tan B$	$\arctan B$																															
4	$(1+B \times 2) \times .5$	$(1+B \times 2) \times .5$																															
5	$\sinh B$	$\operatorname{arcsinh} B$																															
6	$\cosh B$	$\operatorname{arcosh} B$																															
7	$\tanh B$	$\operatorname{artanh} B$																															
$A=B$ $A \neq B$ $A < B$ $A > B$ $A \leq B$ $A \geq B$	Equal Not equal Less than Greater than Not greater than Not less than	Result is 1 if the relation holds, 0 otherwise. $3 \geq 5 \ 6 \ 3 \ 1 \rightarrow 0 \ 0 \ 1 \ 1$																															
$A \wedge B$ $A \vee B$ $A \wedge B$ $A \vee B$	AND OR NAND NOR $A \wedge B \rightarrow \sim A \wedge B$ $A \vee B \rightarrow \sim A \vee B$	Elements of A and B must be 1's or 0's.	<table> <tr> <th>A</th><th>B</th><th>$A \wedge B$</th><th>$A \vee B$</th><th>$A \wedge B$</th><th>$A \vee B$</th></tr> <tr> <td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td></tr> <tr> <td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td></tr> <tr> <td>1</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td></tr> <tr> <td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr> </table>	A	B	$A \wedge B$	$A \vee B$	$A \wedge B$	$A \vee B$	1	1	1	1	0	0	0	1	0	1	1	0	1	0	0	1	1	0	0	0	0	0	1	1
A	B	$A \wedge B$	$A \vee B$	$A \wedge B$	$A \vee B$																												
1	1	1	1	0	0																												
0	1	0	1	1	0																												
1	0	0	1	1	0																												
0	0	0	0	1	1																												

```

                2x2      (Scalar argument and matrix argument.)
2   4   6
8  10  12
14 16 18

                A+A      (Two arguments with identical shapes.)
2   4   6
8  10  12
14 16 18

p(1 1p4)+(1 1 1p3) (The larger rank prevails.)
1   1   1

```

The table at the beginning of this chapter describes most of the functions in complete detail. Most of these functions are familiar mathematical functions or incorporate very simple concepts. Therefore, the discussion below deals with only a few of the less familiar functions or special cases.

FLOOR AND CEILING

The functions floor and ceiling always return an exact integer. The result depends on the value of OCT as follows: If $(\text{B} \text{ NINT } B) \leq \text{OCT} \times (1 + \text{NINT } B)$ the result is $\text{NINT } B$, where $\text{NINT } B$ is the nearest integer to B . Otherwise, the result is the least integer larger than B for ceiling, or the largest integer smaller than B for floor. Note that $B - \text{B}$ can be negative in cases where OCT is not zero and B is slightly less than an integer.

POWER

In keeping with proper mathematics, the power function does not allow taking square roots of negative numbers (e.g., $\sqrt{-1.5}$), but it does allow taking cube roots of negative numbers (e.g., $\sqrt[3]{-1.5}$). To distinguish these cases, the power function attempts to represent the right argument P as a rational number N/M , where N is an integer and M is the least integer such that $(N/M) = P$. Note that $(N/M) = |P|$ depends on OCT . If the left argument is negative and the rational representation has an even denominator, the power function gives a *DOMAIN ERROR*. If the left argument is negative and the rational fraction has an odd denominator, the result is negative if the numerator is odd and is positive if the numerator is even.

RESIDUE

The residue function is slightly more sophisticated than the definition in the table. For example, $2|2-.5 \times \text{OCT}$ would give the improper negative result $-.5 \times \text{OCT}$. The actual algorithm returns zero if $B - A \times \text{B} + A \times A = 0$ would give a result having a sign opposite to the sign of A .

COMBINATIONS-OF

The combinations-of function returns limit values of $A:B$ if A , B , or $B-A$ are negative integers. That is, the result is zero if A , B , and $B-A$ are all negative integers or if B is not a negative integer but either A or $B-A$ is a negative integer.

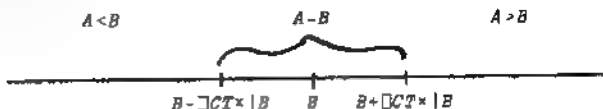
CIRCULAR FUNCTIONS

The domains and ranges of the circular functions are given below.

N	NOB	Domain	Range (-N)OB	Domain	Range
0	$(1-B*2)*.5$	$12 B$	$12Z$		
1	$\sin B$		$12 Z$	$\arcsin B$	$12 B$ $(0.5)2 Z$
2	$\cos B$		$12 Z$	$\arccos B$	$12 B$ $(20)A2\leq 01$
3	$\tan B$			$\arctan B$	$(0.5)2 Z$
4	$(1+B*2)*.5$		$1\leq Z$	$(1+B*2)*.5$	$1\leq B$ $0\leq 2$
5	$\sinh B$		$1\leq Z$	$\operatorname{arcsinh} B$	
6	$\cosh B$		$1\leq Z$	$\operatorname{arcosh} B$	$1\leq B$ $0\leq 2$
7	$\tanh B$		$12 Z$	$\operatorname{artanh} B$	$12 B$

RELATIONAL FUNCTIONS

The functions $=$ and $>$ are the only scalar functions that can be used with arguments of character type. Characters can be compared with numbers, but the result always shows inequality. For numeric A and B , the result for $A=B$ is 1 if $|B-A|$ is not greater than $[CT*|B|$. The three conditions $A=B$, $A > B$, and $A < B$ are always exclusive. For example, if $A=B$ gives 1, then $A > B$ and $A < B$ give 0. The range where two numbers are considered equal is illustrated below:



Note that when B is zero, $A=B$ gives 1 only if A is exactly zero.

Chapter 4. Array Concepts and Indexing

An APL array can be visualized as an arrangement of values along n orthogonal coordinates, where n is 0 to 75 for this particular APL system. The positions along the coordinates are numbered 1, 2, 3, etc. in 1-origin, and they are numbered 0, 1, 2, etc. in 0-origin. The number of elements along a coordinate can be 0 or more. The lengths of the array along the coordinates are called the dimensions of the array, and the number of coordinates is called the rank of the array. The names scalar, vector, and matrix are used to denote arrays of rank 0, 1, and 2, respectively. No special names exist for arrays of rank greater than 2. The APLUM system has an arbitrary limit of 75 as the maximum rank of an array, but in practice, this limit is so large that it is not restrictive. Contrary to common casual practice in mathematics, an APL array has a definite rank--a one-element vector is not the same as a scalar, and a matrix with one row or column is not a vector.

The last coordinate of an array is conventionally considered to be the column coordinate, the second from last coordinate is the row coordinate, and the third from last coordinate is the plane coordinate. The following examples show how various arrays can be formed and displayed:

```
      3      (A scalar.)
3

      14      (A vector.)
1 2 3 4

      2 3n:6   (A matrix.)
1 2 3
4 5 6

      2 3: 'ABCDEF' (A matrix of characters.)
ABC
DEF
```

Summary of Chapter 4.

Function	Description
<code>size</code> Size	Returns a vector containing the dimensions of B . The result has 0 elements for a scalar B , 1 element for a vector, and 2 elements for a matrix.
<code>reshape</code> Reshape	Forms a result having the dimensions specified by the left argument and having elements taken from the right argument in odometer order.
<code>(:)</code> Ravel	The result is a vector containing all elements of B in odometer order.
<code>B[I1:I2, I3; ... , IN]</code> Indexed selection	The result has as dimensions $(pI1), (pI2), (pI3), \dots, (pIN)$ and contains those elements of B for which their first index is in $I1$ and their second index is in $I2$, etc. If a list element is vacant, all possible index values are used.
<code>R[I1:I2:I3; ... ; IN]+B</code> Indexed specification	The indicated elements of R are set to corresponding values from B . Either B must be a one-element array, or the dimensions of B must match $(pI1), (pI2), (pI3), \dots, (pIN)$ except that dimensions of 1 are ignored. If a list element is vacant, all possible index values are used.

```

      2 3 4 124
1     2 3 4
5     6 7 8
9    10 11 12

13 14 15 16
17 18 19 20
21 22 23 24

```

(Two planes, three rows, four columns.)

The last example shows that a rank-3 array is printed as a number of matrices separated by 1 blank line. A rank-4 array would be printed as a number of rank-3 arrays separated by two blank lines, and in general, a rank- N array is displayed as a number of arrays of rank $N-1$ separated by $N-2$ blank lines. An empty array prints as a blank line.

One often visualizes an array as a spatial arrangement of values. The spatial conceptualization leads to use of terms like "shape of array" and "vector along the K th coordinate." These terms are important enough to give precise meanings for them. We define the "shape of an array" to be the result given by the size function (to be discussed in this chapter). As a consequence, a vector and a one-row matrix have different shapes, even though they may be visualized to look the same (and in fact, the system prints them identically). We define "a vector along the K th coordinate" to be a vector of those elements in the array for which the coordinates other than the K th are the same, and the i th element of the vector has i as its K th coordinate in the array--that is, a line of values aligned in the direction of the K th coordinate.

RESHAPE: $R \leftarrow VpB$

The reshape function was used in some of the previous examples to form arrays. The function forms a result having the dimensions specified by the vector (or scalar) left argument and having elements taken from the right argument. Elements are taken in first to last order, and if they are exhausted, they are used again beginning with the first. The right argument must not be empty unless the result will be empty--"reshape never makes something out of nothing."

ORDERING OF ELEMENTS

The elements of an array are considered to be ordered. The reshape function takes elements according to this ordering. The ordering is the same as the order in which the elements are printed by a terminal. The order is called odometer order because the indices (coordinate positions) vary in the same way as the digits of an odometer. For example, for an array $A3$ having dimensions 2 3 4 the elements in odometer order are:

```

A3[1;1;1]
A3[1,1,2]
A3[1;1;3]
A3[1;1,4]
A3[1;2;1]
A3[1;2;2]
.
.
A3[2;3;4]

```

SIZE: R+pB

The size function returns a vector of the dimensions of its right argument. Because there is one element in the result for each dimension of B, the result has 0 elements for a scalar B, 1 element for a vector, 2 elements for a matrix, and so forth. Note that because pB has one element for each dimension of B, ppB gives the rank of B as a one-element vector. The following examples illustrate the size function for arrays of various ranks:

```

p3      (A scalar.)
        (A blank line indicates an empty vector
        result.)

pp3

0

p13     (A vector.)

3

pp13

1

p2 3p16 (A matrix)

2 3

pp2 3p16

2

p2 3 5p130 (A rank-3 array.)

2 3 5

pp 2 3 5p130

3

```

RAVEL: Z+,B

The ravel function returns a vector result containing all the elements of the right argument in odometer order. For example:

```

.2 3p16 (Changing a matrix to a vector.)
1 2 3 4 5 6

```

9.3 (Changing a scalar to a vector.)

1

The `ravel` function can be used to determine the number of elements in an arbitrary array. The number of elements in `B` is `p.B` (Note that the `ravel` function could be omitted in this expression if `B` were always a vector.)

2.1
2.2

INDEXED SELECTION: `R=B[I1;I2;I3; ... ;IN]`

Indexed selection chooses those elements of an array for which all indexes occur in the respective list elements. For example, if `M` is a matrix, `M[3;4]` gives the element having 3 as its row index and 4 as its column index. Similarly, `M[2 3;4 5]` gives those elements in the second and third rows that are also in the fourth and fifth columns. If a list element is vacant, `IN` is used, where `N` is the length along that coordinate. The index values must be integers in the range of coordinates of elements in `B`. The index list for an array of rank `K` must have `K-1` semicolons. The result `R` has the dimensions `(pI1),(pI2),(pI3), ... ,(pIN)`. Hence the rank of `R` is the sum of the ranks of the indices. If the indices are vectors, the result satisfies

`R[K1;K2;K3; ... ;KN]=B[I1[K1];I2[K2];I3[K3]; ... ;IN[KN]]`

When the indices are not all vectors, the result is:

`((pI1),(pI2),(pI3), ... ,(pIN))pB[,I1;.I2;.I3; ... ;.IN]`

Indexed selection cannot be applied to a scalar. The following examples show indexed selection applied to vectors and matrices:

2.2
2.3
2.4
2.5

`V←3 5 9 12`
`V[1]`

3
`V[4]`

12
`V[5]` (An error results from a request
07: INDEX ERROR for an element that does not exist.)

`V[5]`
/ `V[5,6]` (Because `V` is a vector, its rank is
06: RANK ERROR incompatible with the index list.)
`V[5;6]`
/

```

      V[1 2 1 1 2]
3 6 3 3 3

      [M+3 4p:12
1 2 3 4
5 6 7 8
9 10 11 12
      N[2:3]
7
      N[2; ]      (Row 2, all columns.)
5 6 7 8
      N[:3]      (All rows, column 3.)
3 7 11
      N[2:3 4]
7 8
      N[1 2 1;3 1]
3 1
7 5
3 1

      [M+3 5p2 3 2 3 + 2 3 5 3 2 1 3 1 3 1
2 3 2 3 4
2 3 5 3 2
1 3 1 3 1

      ' _[QX'[X]
1 |Q
|[X]_      (A matrix of characters.)
: |

```

INDEXED SPECIFICATION: $R[I_1;I_2;I_3; \dots ;I_N]-B$

Indexed specification allows setting of selected elements of R . The index list indicates elements to be set in the same way as for indexed selection (see previous section). The restrictions on list elements are also the same as for indexed selection. The array B must be a scalar (or one-element array) or must have dimensions $(pI_1), (pI_2), (pI_3), \dots, (pI_N)$ except that dimensions of length 1 are ignored in the comparison. If B is not a scalar (or one-element array), the elements of B are taken in odometer order and placed in appropriate locations in R . If two elements of B are placed in the same position in R , the last one in odometer order in B prevails. Both R and B must be of the same type (i.e., character or numeric). The shape of R is not changed by the operation. R must not be a scalar.

```

      V+3 6 9
      V[2]+-1
      V
3 -1 9

```

```

      V[2 3]←10 12
      ✓
3 10 12

      V[3 3]←15 16
      V
3 10 16

      M←M+2 30 '*'
***

      M[1;1 2 3]←'0' (A scalar is used repeatedly.)
      M
000
***

      M[:,1]←'0' (All rows, column 1.)
      M
000
0**

      M[1;2 3]←'+* '
      M
0+*
0**

```


Chapter 5. Mixed Functions

The class of mixed functions includes all functions that are not system functions, composite functions, or scalar functions. Because few patterns exist between the mixed functions, they must be discussed individually to describe the arguments they allow and the results they produce. Chapter 4 already discussed the three mixed functions `reshape`, `size`, and `ravel`.

EXCEPTION RULES

Most of the mixed functions have "normal" cases for which the results are relatively simple to express in terms of the arguments. They also generally have additional special cases that are convenient but are treated as exceptions. The following are some of the reasons these exceptions are allowed:

Exceptions to overcome notational difficulty. There is no way to represent an empty numeric vector constant in an expression, and `10` is inconvenient to use as a left argument because it must be surrounded by parentheses. Hence `'pB` is allowed in place of `(10)pB`. However, the only other case where an empty character argument is allowed where a nonempty character argument would not be is the `catenate` function. (However, the system functions `LISTOP`, `TRACE`, and `EXTIME` also allow empty character left arguments.) Another class of exceptions to overcome notational difficulty arises because it is not possible to type a one-element vector constant. Because a constant consisting of a single character or number is a scalar, many functions allow a scalar in place of a one-element vector. However, the left argument for `index-of` and the arguments to `grade up` and `grade down` are not allowed to be scalars.

Summary of Mixed Functions Presented in Chapter 5.

Function	Description, Examples
$\downarrow B$ Index generator	Produces a vector of the first B integers. $\downarrow 5 \rightarrow 1\ 2\ 3\ 4\ 5$
$V \downarrow B$ Index-of	For each element of B gives the first index in the vector V where the element is found or $\downarrow \rightarrow V$ (in 1-origin) if the element is absent from V . $5\ 6\ 7\ 8 \downarrow 6\ 5\ 2 \rightarrow 2\ 1\ 5$
$A \in B$ Membership	Returns 1 for each element of A that occurs in B and returns 0 for other elements of A . $1\ 3\ 5 \in 2\ 3 \rightarrow 0\ 1\ 0$
$S1?S2$ Deal	Chooses $S1$ random numbers from $\downarrow S2$ without any duplications.
$\downarrow V$ Grade up	The i th element of the vector result is the index in V of the i th smallest value in V . $V[\downarrow V]$ gives V sorted in increasing order. $\downarrow 3.3\ 5.2\ 1.1 \rightarrow 3\ 1\ 2$
$\uparrow V$ Grade down	The i th element of the vector result is the index in V of the i th largest value in V . $V[\uparrow V]$ gives V sorted in decreasing order.
$A, [K]B$ Join	Joins A and B along the K th coordinate. $1\ 2\ 3, 4\ 5 \rightarrow$ $1\ 2\ 3\ 4\ 5$
$V/[K]B$ Compress	The result includes elements along the K th coordinate of B for which there are corresponding 1's in V and does not include elements for which there are 0's in V . $1\ 0\ 1/1\ 2\ 3 \rightarrow 1\ 3\ \quad 1\ 0\ 1/'ABC' \rightarrow 'AC'$

Summary of Mixed Functions Presented in Chapter 5,
Continued.

Function	Description, Examples
$V[K]B$ Expand	Expands by inserting zeros (if B is numeric) or blanks (if B is of character type) where there are 0's in V and selects consecutive elements along the K th coordinate of B where there are 1's in V . 1 0 1 0\3 4→3 0 4 0 1 0 1\AB'→A B'
$A+B$ Take	Selects the first (if $A[K]>0$) or last (if $A[K]<0$) $ A[K] $ elements along the K th coordinate of B . If $ A[K] $ exceeds $(\rho B)[K]$, zeros or blanks are used as the extra elements. 3+1 2 3 4 5→1 2 3 -3+ABCD E'→CDE' 4+1 2→1 2 0 0
$A+B$ Drop	Drops the first (if $A[K]>0$) or last (if $A[K]<0$) $ A[K] $ elements along the K th coordinate of B . If $ A[K] $ exceeds $(\rho B)[K]$, the K th dimension of the result is zero. 3+1 2 3 4 5→4 5 3+ABCD E'→AB'
$\phi[K]B$ Reverse	Reverses the order of elements along the K th coordinate of B . 45 6 7→7 6 5 ϕ 'ABCD'→DCBA'
$A\phi[X]B$ Rotate	Shifts vectors along the K th coordinate of B in a negative direction (for $A>0$) or positive direction (for $A<0$). 241 2 3 4 5→3 4 5 1 2 24'ABCDE'→DEABC'
ρB Monadic transpose	Reverses coordinates of B . $\rho\rho B \rightarrow \phi\rho B$
$A\phi B$ Dyadic transpose	Interchanges coordinates of B according to A . The K th coordinate of the result corresponds to the $(A+K)/\rho A$ coordinate of B .

Summary of Fixed Functions Presented in Chapter 5, Continued.

Function	Description, Examples
$A \vdash B$ Base value	Evaluates B as a number represented in a number system having radices A . 2 2 2.1 0 1 \vdash 5 10 10 10 12 3 4 \vdash 234
$A \vdash B$ Represent	Represents B in the number system having radices A . 2 2 2 \vdash 5 \vdash 1 0 1 10 10 10 \vdash 296 \vdash 2 9 6
$\$B$ Execute	Executes the character vector B as an APL statement. \$ '15' \vdash 1 2 3 4 5
$\vee B$ Monadic format	Produces a character array representation of B . Except for treatment of lines longer than $\square PW$, $\vee B$ looks exactly like B when printed.
$A \vee B$ Dysdic format	Represents columns of B according to the format specified by pairs of numbers on A . The first element of a pair in A is the width of the field (0 to have the system choose a width), and the second element of the pair gives the number of digits beyond the decimal if positive. If the second element of the pair is negative, its absolute value determines the total number of digits, and exponential format is used.
$\@B$ Matrix inverse	Matrix inverse of B . Same as $I \@ B$ where I is an identity matrix.
$A \@ B$ Matrix divide	Solution to a system of equations (for a square matrix B) or least squares regression coefficients (if B has more rows than columns). Same as $(\@ B) \vee . * A$.

Exceptions to ignore dimensions of 1. At times it is convenient to treat a row or column of an array as a vector, while at other times it is more convenient to treat it as a matrix. Consequently, some flexibility has been built into functions to allow extra or missing dimensions of 1.

Generalized scalar extension. The dyadic scalar functions allow a scalar argument to be used repeatedly with all elements of the other argument. More generally, some mixed functions allow a single vector, plane, etc. to be used repeatedly with parts of the other argument.

ARRAY TYPES

An array, even if it is empty, is either of character type or numeric type. Those mixed functions that rearrange elements of an array or select elements of an array always return a result having the same type as the right argument. For example, $0p\ 'ABCD'$ gives an empty result of character type.

AXIS OPERATORS

For several of the mixed functions (and composite functions) an axis operator can be used to specify the coordinate along which the operation is to be performed. If no axis is specified, the last coordinate is assumed. Alternate symbols can be used to perform the operations along the first coordinate. These forms are:

<u>Last coordinate</u>	<u>First coordinate</u>	<u>Kth coordinate</u>
$A\ B$	$A\uparrow B$	$A\,[K]B$
$A\ /B$	$A\uparrow/B$	$A\,[K]/B$
$A\ \backslash B$	$A\uparrow\backslash B$	$A\,[K]\backslash B$
ϕB	$\phi\uparrow B$	$\phi\,[K]B$
$A\ \phi B$	$A\ \phi\uparrow B$	$A\ \phi\,[K]B$

Note that the symbols for performing the operations along the first coordinate are not allowed to be used with an axis operator. For example, $\phi[K]B$ would produce a *SYNTAX ERROR*.

The value used for an axis operator must be a one-element array, and for functions other than join, it must be an integer in $1ppB$ (except that if B is a scalar, it may be $^{-1+1}$). For the join function (e.g., $A\,[K]B$) the value of K may be an integer in $1(ppA)(ppB)+1$ or a half integer obtained by adding or subtracting .5 from one of those integers.

INDEX GENERATOR: $p+1B$

The index generator function produces a vector of length B containing the first B integers. The result depends on the current origin.

Requirements for B . B must be a one-element array containing a nonnegative integer.

Examples.

```
13 (In 1-origin.)
1 2 3

0 10 0
15
0 1 2 3 4 (In 0-origin.)

10
(Blank line indicates 10 is empty.)
```

INDEX-OF: $R+V, B$

The index-of function returns for each element of B the least index I in the vector V for which $V[I]$ equals the element of B . If no value in V is equal, the result element is $1+pV$ in 1-origin, or pV in 0-origin. The comparisons use ECF so that elements of V and B may be considered equal even if they differ slightly.

Requirements for V and B . V must be a vector--a scalar is not allowed. B may be of any shape and the result will have that shape.

Examples.

```
4 5 6 2 5
4 2

D+M-2 3p 'DEFGHI'
DEP
GHI

'HIDE DOG' 1M
3 4 9 (A matrix result for a matrix right argument.)
0 2 2

7 8 9 1 'AB'
4 4 4 (Characters never equal numbers.)
```

```

      'ABA'⊆'ABAB'
1 2 1 2

      ⓁIO←3
      'ABA'⊆'ABAB'
0 1 0 1      (The 0-origin result is 1 less.)

```

MEMBERSHIP: $B \leftarrow A \in B$

The membership function returns 1 for each element of A that occurs in B . For numeric arguments the comparisons use the current value of DCT , so values may differ slightly and still be considered equal.

Requirements for A and B . A and B may have any shape. The result has the same shape as A .

Examples.

```

      1 2 3⊆3 1 5 4 9
1 0 1

      'ABCD'⊆'BACKS'
1 1 1 0

      ⓁA+2 3⊆'CATDOG'
CAT
DOG

      A⊆'GOAT'
0 1 1
0 1 1      (The result has the shape of the left argument.)

      'GOAT'⊆A
1 1 1 1

      'ABC'⊆1 2 3 4
0 0 0

```

DEAL: $R \leftarrow S1 \text{ } S2$

The deal function chooses at random $S1$ values from $S2$ without repetitions.

Requirements for $S1$ and $S2$. Both $S1$ and $S2$ must be one-element arrays containing nonnegative integers such that $S1 \leq S2$. The result is a vector of length $S1$.

Examples.

```
      375
3 1 4

      375
4 5 3

      575
1 2 5 3 4

      070+0
      575
3 2 0 1 2   (0-origin.)
```

GRADE UP AND GRADE DOWN: $R \leftarrow \#B$ and $R \leftarrow \#B$

The i th element of the vector result R is the index in B where the i th smallest (for grade up) or the i th largest (for grade down) element of B occurs. The comparisons do not use OCT. If a value occurs more than once in B , the indices of those values occur together in R in increasing order.

Requirements for B . B must be a numeric vector. The result R is a numeric vector of the same length as B .

Examples.

```
      #3.3 1.1 2.2 4.4 1.1 5.5
2 5 3 1 4 5

      #3.3 1.1 2.2 4.4 1.1 5.5
6 4 1 3 2 5

      V+3.3 1.1 2.2 4.4 1.1 5.5
      V[#V] (To sort in increasing order.)
1.1 1.1 2.2 3.3 4.4 5.5

      V[#V] (To sort in decreasing order.)
5.5 4.4 3.3 2.2 1.1

      P+3 4 5 1 2
      ('ABCD'[P])[#P] (#P is the inverse of a permutation
ABCDP vector P.)

      X+'ABC'
      Y+'DEF'
      Z+'GHI'
      (X,Y,Z)[# 0 2 1 1 2 0 2 1]
AODENBCIF (Select next from X for a 0, Y for a 1,
           Z for a 2.)
```

NO=0

43.3 1.1 2.2

1 2 0 (0 origin.)

JOIN: R+A,[K]B

The join function connects A and B along a coordinate already existing in A or B or along a new coordinate of length 1 inserted into each. The first elements along the coordinate come from A and the rest come from B . When K is an integer, the operation is called catenate. When K is not an integer, the operation is called laminate and the new coordinate of length 1 is inserted into each argument between the existing K coordinate and $[K]$ coordinate.

Requirements for A and B . Except for the special cases below, A and B must have the same rank, and dimensions other than the K th must be the same; that is, $(K=1:pA)/pA$ and $(K=1:pB)/pB$ must be the same. The types of A and B must be the same unless one or both are empty arrays. (Warning: some API systems do not allow empty arrays to have a different type. It is recommended that differing types be avoided for compatibility.) The shape of the result is the same as the shape of the two arguments except that the K th coordinate of the result is $(pA)[K]+(pB)[K]$. If both arguments are empty and of differing types, the result is numeric.

Exception cases. If A or B is a scalar (but not both), it is reshaped to have the shape of the other argument except that the K th dimension is 1 for catenate. If both arguments are scalars, they are treated as one-element vectors for catenate. For catenate, one argument may have a rank 1 less than the rank of the other argument. In this case a new coordinate of length 1 is inserted to become the K th.

Examples

```
      1 2 3,4 5 6      (Joining two vectors.)
1 2 3 4 5 6
```

```
Q=I+2 3p 'x'
```

```
***
***
```

```
Q=I+3 3p '0'
```

```
ooo
ooo
ooo
```

```

      M,[1]N
***
***
000
000
---

    0+L-2 4p'0'
0000
0000

      M,L
***0000
***0000

      N,'+'
***+
***+
      (The scalar is treated as a one-column
      matrix.)
      N,'34'
      (A vector is treated as a one-column
      matrix.)
***3
***4

      N,[1]'345' (A vector is treated as a one-row matrix.)
***
***
345

      1 2 3,[.5]4 5 6 (Laminate along a new first coordinate.)
1 2 3
4 5 6

      1 2 3,[1.5]4 5 6 (Laminate along a new last coordinate.)
1 4
2 5
3 6

      1 2 3,[1.5]4
1 4
2 4
3 4

```

COMPRESS: $R=V/[K]B$

The compress function shortens B along the K th coordinate by omitting those elements for which there are corresponding 0's in V .

Requirements for V and B . V must be a vector and all elements of V must be 1's or 0's. The length of V must be the same as $(\rho B)[K]$. The result has the same dimensions as B except that the K th dimension is $+V$.

Exception cases. If V or B is a scalar it is treated as a one element vector. Then if V is a one-element vector, it is extended to the length of B along the K th coordinate. If B is a one-element vector, it is extended to the length of V .

Examples.

```
      1 0 1 0 1/1 2 3 4 5
1 3 5
```

```
      1 0 1 0 1/'ABCDE'
ACE
```

```
      1/'ABCDE'
ABCDE
```

```
      0/'ABCDE'
```

(Blank line indicates an empty result.)

```
      (+M+3 4p112
1 2 3 4
5 6 7 8
9 10 11 12
```

```
      1 0 1 1/M
1 3 4
5 7 8
9 11 12
```

```
      1 0 1/[1]M      (Same as 1 0 1/M.)
1 2 3 4
9 10 11 12
```

```
      1 0 1/4
4 4
```

EXPAND: $R+V\backslash[K]B$

The result is formed by expanding B along the K th coordinate by filling with zeros (if B is numeric) or blanks (if B is of character type) in those positions in R for which there are corresponding 0's in V .

Requirements for V and B . Ignoring the special cases, V must be a vector containing only 1's and 0's such that $(+/V)=(0B)[K]$. The result R has the same dimensions as B except that the K th dimension is αV .

Exception cases. If V or B is a scalar, it is treated as a one-element vector.

Examples.

```

      1 3 1 0 1\ 2 3
1 3 2 0 3

      [M+2 3p:6
1 2 3
4 5 6

      1 0 1 0 1\M
1 0 2 0 3
4 0 5 0 6

      1 0 1\1]M (Same as 1 0 1\M.)
1 2 3
0 0 0
4 5 6

      p1/2 (A vector result.)
1

      ' '=0\'' (An empty array can be expanded.)
1

      0\10
0

```

TAKE: $R \leftarrow V \uparrow B$

The take function selects $|V[K]|$ first elements (for $V[K]>0$) or last elements (for $V[K]<0$) along the K th coordinate of B . If $|V[K]|$ exceeds $(\rho B)[K]$, zeros (if B is numeric) or blanks (if B is of character type) are used to provide the extra elements.

Requirements for V and B . Ignoring the special cases below, V must be a vector having an integer for each dimension of B . That is, $(\rho V) = \rho \rho B$. The result R has dimensions $|V|$.

Special cases. If V is a scalar, it is treated as a one-element vector. If B is a scalar, it is treated as a one-element array of rank ρV .

Examples.

```

      3+1 2 3 4 5
1 2 3

      ~3+1 2 3 4 5
3 4 5

```

```

      3+'ABCDE'
A2C

      5+1 2 3
1 2 3 0 0

      0-M+3 4p12
1 2 3 4
5 6 7 8
9 10 11 12

      2 -5+M (First 2 rows, last 5 columns.)
0 1 2 3 4
0 5 6 7 8

      3+1 0 (Take can be applied to an empty array.)
0 0 0

      2 3+5 (5 is treated as a 1 by 1 matrix.)
5 0 0
0 0 0

```

DROP: $R+V+B$

The drop function forms its result by omitting $|V[K]|$ first elements (if $V[K]>0$) or last elements (if $V[K]<0$) along the K th coordinate of B .

Requirements for V and B . Ignoring the special cases below, V must be a vector of integers, and pV must be the same as pB . The result has dimensions $0[(pB)-|V|]$.

Special cases. If V is a scalar, it is treated as a one-element vector. If B is a scalar, it is treated as a one-element array of rank pV .

Examples.

```

      3+1 2 3 4 5
4 5

      -2+'ABCDEF'
ABCD

      10+1 2 3
      (Blank line indicates an empty result.)

      0-M+3 4p12
1 2 3 4
5 6 7 8
9 10 11 12

```

```

1 2 3 4 5 6 (First row and last 2 columns are dropped.)
5 6
9 10

20 0+3
1 1 (The scalar was treated as a matrix.)

(10)+3
3

```

REVERSE: $R \leftarrow \phi[K]B$

The reverse function reverses the order of elements along the K th coordinate of B . The result has exactly the same shape as B .

Examples.

```

      1 2 3 4 5 6
6 5 4 3

```

```

      1 2 3 4 5 6
      7 8 9 10 11 12
      13 14 15 16 17 18
      19 20 21 22 23 24
      25 26 27 28 29 30
      31 32 33 34 35 36
      37 38 39 40 41 42
      43 44 45 46 47 48
      49 50 51 52 53 54
      55 56 57 58 59 60
      61 62 63 64 65 66
      67 68 69 70 71 72
      73 74 75 76 77 78
      79 80 81 82 83 84
      85 86 87 88 89 90
      91 92 93 94 95 96
      97 98 99 100 101 102
      103 104 105 106 107 108
      109 110 111 112 113 114
      115 116 117 118 119 120
      121 122 123 124 125 126
      127 128 129 130 131 132
      133 134 135 136 137 138
      139 140 141 142 143 144
      145 146 147 148 149 150
      151 152 153 154 155 156
      157 158 159 160 161 162
      163 164 165 166 167 168
      169 170 171 172 173 174
      175 176 177 178 179 180
      181 182 183 184 185 186
      187 188 189 190 191 192
      193 194 195 196 197 198
      199 200 201 202 203 204
      205 206 207 208 209 210
      211 212 213 214 215 216
      217 218 219 220 221 222
      223 224 225 226 227 228
      229 230 231 232 233 234
      235 236 237 238 239 240
      241 242 243 244 245 246
      247 248 249 250 251 252
      253 254 255 256 257 258
      259 260 261 262 263 264
      265 266 267 268 269 270
      271 272 273 274 275 276
      277 278 279 280 281 282
      283 284 285 286 287 288
      289 290 291 292 293 294
      295 296 297 298 299 300
      301 302 303 304 305 306
      307 308 309 310 311 312
      313 314 315 316 317 318
      319 320 321 322 323 324
      325 326 327 328 329 330
      331 332 333 334 335 336
      337 338 339 340 341 342
      343 344 345 346 347 348
      349 350 351 352 353 354
      355 356 357 358 359 360
      361 362 363 364 365 366
      367 368 369 370 371 372
      373 374 375 376 377 378
      379 380 381 382 383 384
      385 386 387 388 389 390
      391 392 393 394 395 396
      397 398 399 400 401 402
      403 404 405 406 407 408
      409 410 411 412 413 414
      415 416 417 418 419 420
      421 422 423 424 425 426
      427 428 429 430 431 432
      433 434 435 436 437 438
      439 440 441 442 443 444
      445 446 447 448 449 450
      451 452 453 454 455 456
      457 458 459 460 461 462
      463 464 465 466 467 468
      469 470 471 472 473 474
      475 476 477 478 479 480
      481 482 483 484 485 486
      487 488 489 490 491 492
      493 494 495 496 497 498
      499 500 501 502 503 504
      505 506 507 508 509 510
      511 512 513 514 515 516
      517 518 519 520 521 522
      523 524 525 526 527 528
      529 530 531 532 533 534
      535 536 537 538 539 540
      541 542 543 544 545 546
      547 548 549 550 551 552
      553 554 555 556 557 558
      559 560 561 562 563 564
      565 566 567 568 569 570
      571 572 573 574 575 576
      577 578 579 580 581 582
      583 584 585 586 587 588
      589 590 591 592 593 594
      595 596 597 598 599 600
      601 602 603 604 605 606
      607 608 609 610 611 612
      613 614 615 616 617 618
      619 620 621 622 623 624
      625 626 627 628 629 630
      631 632 633 634 635 636
      637 638 639 640 641 642
      643 644 645 646 647 648
      649 650 651 652 653 654
      655 656 657 658 659 660
      661 662 663 664 665 666
      667 668 669 670 671 672
      673 674 675 676 677 678
      679 680 681 682 683 684
      685 686 687 688 689 690
      691 692 693 694 695 696
      697 698 699 700 701 702
      703 704 705 706 707 708
      709 710 711 712 713 714
      715 716 717 718 719 720
      721 722 723 724 725 726
      727 728 729 730 731 732
      733 734 735 736 737 738
      739 740 741 742 743 744
      745 746 747 748 749 750
      751 752 753 754 755 756
      757 758 759 760 761 762
      763 764 765 766 767 768
      769 770 771 772 773 774
      775 776 777 778 779 780
      781 782 783 784 785 786
      787 788 789 790 791 792
      793 794 795 796 797 798
      799 800 801 802 803 804
      805 806 807 808 809 810
      811 812 813 814 815 816
      817 818 819 820 821 822
      823 824 825 826 827 828
      829 830 831 832 833 834
      835 836 837 838 839 840
      841 842 843 844 845 846
      847 848 849 850 851 852
      853 854 855 856 857 858
      859 860 861 862 863 864
      865 866 867 868 869 870
      871 872 873 874 875 876
      877 878 879 880 881 882
      883 884 885 886 887 888
      889 890 891 892 893 894
      895 896 897 898 899 900
      901 902 903 904 905 906
      907 908 909 910 911 912
      913 914 915 916 917 918
      919 920 921 922 923 924
      925 926 927 928 929 930
      931 932 933 934 935 936
      937 938 939 940 941 942
      943 944 945 946 947 948
      949 950 951 952 953 954
      955 956 957 958 959 960
      961 962 963 964 965 966
      967 968 969 970 971 972
      973 974 975 976 977 978
      979 980 981 982 983 984
      985 986 987 988 989 990
      991 992 993 994 995 996
      997 998 999 1000 1001 1002
      1003 1004 1005 1006 1007 1008
      1009 1010 1011 1012 1013 1014
      1015 1016 1017 1018 1019 1020
      1021 1022 1023 1024 1025 1026
      1027 1028 1029 1030 1031 1032
      1033 1034 1035 1036 1037 1038
      1039 1040 1041 1042 1043 1044
      1045 1046 1047 1048 1049 1050
      1051 1052 1053 1054 1055 1056
      1057 1058 1059 1060 1061 1062
      1063 1064 1065 1066 1067 1068
      1069 1070 1071 1072 1073 1074
      1075 1076 1077 1078 1079 1080
      1081 1082 1083 1084 1085 1086
      1087 1088 1089 1090 1091 1092
      1093 1094 1095 1096 1097 1098
      1099 1100 1101 1102 1103 1104
      1105 1106 1107 1108 1109 1110
      1111 1112 1113 1114 1115 1116
      1117 1118 1119 1120 1121 1122
      1123 1124 1125 1126 1127 1128
      1129 1130 1131 1132 1133 1134
      1135 1136 1137 1138 1139 1140
      1141 1142 1143 1144 1145 1146
      1147 1148 1149 1150 1151 1152
      1153 1154 1155 1156 1157 1158
      1159 1160 1161 1162 1163 1164
      1165 1166 1167 1168 1169 1170
      1171 1172 1173 1174 1175 1176
      1177 1178 1179 1180 1181 1182
      1183 1184 1185 1186 1187 1188
      1189 1190 1191 1192 1193 1194
      1195 1196 1197 1198 1199 1200
      1201 1202 1203 1204 1205 1206
      1207 1208 1209 1210 1211 1212
      1213 1214 1215 1216 1217 1218
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      1243 1244 1245 1246 1247 1248
      1249 1250 1251 1252 1253 1254
      1255 1256 1257 1258 1259 1260
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      1327 1328 1329 1330 1331 1332
      1333 1334 1335 1336 1337 1338
      1339 1340 1341 1342 1343 1344
      1345 1346 1347 1348 1349 1350
      1351 1352 1353 1354 1355 1356
      1357 1358 1359 1360 1361 1362
      1363 1364 1365 1366 1367 1368
      1369 1370 1371 1372 1373 1374
      1375 1376 1377 1378 1379 1380
      1381 1382 1383 1384 1385 1386
      1387 1388 1389 1390 1391 1392
      1393 1394 1395 1396 1397 1398
      1399 1400 1401 1402 1403 1404
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      1597 1598 1599 1600 1601 1602
      1603 1604 1605 1606 1607 1608
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      1837 1838 1839 1840 1841 1842
      1843 1844 1845 1846 1847 1848
      1849 1850 1851 1852 1853 1854
      1855 1856 1857 1858 1859 1860
      1861 1862 1863 1864 1865 1866
      1867 1868 1869 1870 1871 1872
      1873 1874 1875 1876 1877 1878
      1879 1880 1881 1882 1883 1884
      1885 1886 1887 1888 1889 1890
      1891 1892 1893 1894 1895 1896
      1897 1898 1899 1900 1901 1902
      1903 1904 1905 1906 1907 1908
      1909 1910 1911 1912 1913 1914
      1915 1916 1917 1918 1919 1920
      1921 1922 1923 1924 1925 1926
      1927 1928 1929 1930 1931 1932
      1933 1934 1935 1936 1937 1938
      1939 1940 1941 1942 1943 1944
      1945 1946 1947 1948 1949 1950
      1951 1952 1953 1954 1955 1956
      1957 1958 1959 1960 1961 1962
      1963 1964 1965 1966 1967 1968
      1969 1970 1971 1972 1973 1974
      1975 1976 1977 1978 1979 1980
      1981 1982 1983 1984 1985 1986
      1987 1988 1989 1990 1991 1992
      1993 1994 1995 1996 1997 1998
      1999 2000 2001 2002 2003 2004
      2005 2006 2007 2008 2009 2010
      2011 2012 2013 2014 2015 2016
      2017 2018 2019 2020 2021 2022
      2023 2024 2025 2026 2027 2028
      2029 2030 2031 2032 2033 2034
      2035 2036 2037 2038 2039 2040
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      2047 2048 2049 2050 2051 2052
      2053 2054 2055 2056 2057 2058
      2059 2060 2061 2062 2063 2064
      2065 2066 2067 2068 2069 2070
      2071 2072 2073 2074 2075 2076
      2077 2078 2079 2080 2081 2082
      2083 2084 2085 2086 2087 2088
      2089 2090 2091 2092 2093 2094
      2095 2096 2097 2098 2099 2100
      2101 2102 2103 2104 2105 2106
      2107 2108 2109 2110 2111 2112
      2113 2114 2115 2116 2117 2118
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      2125 2126 2127 2128 2129 2130
      2131 2132 2133 2134 2135 2136
      2137 2138 2139 2140 2141 2142
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      2173 2174 2175 2176 2177 2178
      2179 2180 2181 2182 2183 2184
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      2191 2192 2193 2194 2195 2196
      2197 2198 2199 2200 2201 2202
      2203 2204 2205 2206 2207 2208
      2209 2210 2211 2212 2213 2214
      2215 2216 2217 2218 2219 2220
      2221 2222 2223 2224 2225 2226
      2227 2228 2229 2230 2231 2232
      2233 2234 2235 2236 2237 2238
      2239 2240 2241 2242 2243 2244
      2245 2246 2247 2248 2249 2250
      2251 2252 2253 2254 2255 2256
      2257 2258 2259 2260 2261 2262
      2263 2264 2265 2266 2267 2268
      2269 2270 2271 2272 2273 2274
      2275 2276 2277 2278 2279 2280
      2281 2282 2283 2284 2285 2286
      2287 2288 2289 2290 2291 2292
      2293 2294 2295 2296 2297 2298
      2299 2300 2301 2302 2303 2304
      2305 2306 2307 2308 2309 2310
      2311 2312 2313 2314 2315 2316
      2317 2318 2319 2320 2321 2322
      2323 2324 2325 2326 2327 2328
      2329 2330 2331 2332 2333 2334
      2335 2336 2337 2338 2339 2340
      2341 2342 2343 2344 2345 2346
      2347 2348 2349 2350 2351 2352
      2353 2354 2355 2356 2357 2358
      2359 2360 2361 2362 2363 2364
      2365 2366 2367 2368 2369 2370
      2371 2372 2373 2374 2375 2376
      2377 2378 2379 2380 2381 2382
      2383 2384 2385 2386 2387 2388
      2389 2390 2391 2392 2393 2394
      2395 2396 2397 2398 2399 2400
      2401 2402 2403 2404 2405 2406
      2407 2408 2409 2410 2411 2412
      2413 2414 2415 2416 2417 2418
      2419 2420 2421 2422 2423 2424
      2425 2426 2427 2428 2429 2430
      2431 2432 2433 2434 2435 2436
      2437 2438 2439 2440 2441 2442
      2443 2444 2445 2446 2447 2448
      2449 2450 2451 2452 2453 2454
      2455 2456 2457 2458 2459 2460
      2461 246
```

Requirements for A and B . Ignoring the exception below, A must have one element for each vector in B along the k th coordinate. That is, ρA must be $(K \div \rho B) \div \rho B$. Thus the dimensions of A must be like those of B except that the k th dimension of B is absent from A . The result has the same shape as B .

Special cases. If A is a scalar, it is extended to become an array having dimensions suitable for B . Rotation of a scalar is allowed, but the left argument must be a scalar, and the result is the same as B .

Examples.

```
      2 0 1 2 3 4 5 (Rotation by 2 positions to the left.)
3 4 5 1 2
```

```
      2 0 1 2 3 4 5 (Rotation by 2 positions to the right.)
4 5 1 2 3
```

```
      2 0 'ABCDE'
CDEAB
```

```
      0 1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4
5 6 7 8
9 10 11 12
```

```
      0 1 2 3 4
1 2 3 4      (Rows are shifted.)
5 6 7 8
11 12 3 10
```

```
      0 1 2 3 4 5 6 7 8 9 10 11 12 (Same as 0 1 1 2 0 1 1 2 0 1 1 2 0 1 1 2)
1 10 7 12
5 2 11 4
9 6 3 8
```

```
      1 0 1 2 3 4 5 6 7 8 9 10 11 12
2 3 4 1
6 7 8 5
10 11 12 9      (All rows are shifted by 1.)
```

MONADIC TRANSPOSE: $R \rightarrow \rho B$

The monadic transpose function reverses the coordinates in B . Thus the last coordinate in R corresponds to the first in B , the second to the last corresponds to the second in B , and so forth. For a vector or scalar, the result is the same as the argument. For a matrix, the result is the usual matrix transpose. For an array of rank 3, $R[i,j,z]$ is the same as $B[x,j,i]$. The shape of the result is $\phi \rho B$.

Examples.

```

      [ ]+M+3 4p12
2  2  3  4
5  6  7  8
9 10 11 12

```

```

      4p12
1  5  9
2  6 10
3  7 11
4  8 12

```

```

      [ ]+C+3 4p 'FOURFIVEFORT'
FOUR
FIVE
FORT

```

```

      4C
PFP
OIO
CVN
RET

```

```

      [ ]+R3+2 3 4p124
1  2  3  4
5  6  7  8
9 10 11 12

```

```

13 14 15 16
17 18 19 20
21 22 23 24

```

```

      p4R3
4 3 2

```

```

      4R3
1  3
5 17
9 21

2 14
6 16
10 22

```

```

3 15
7 19
11 23

```

```

4 16
8 20
12 24

```

DYADIC TRANSPOSE: $B \leftarrow V \otimes B$

The dyadic transpose function interchanges coordinates of B according to the integer values in the vector V .

Requirements for V and B . Ignoring the special case below, V must be a vector having one element for each dimension of B --that is, V and B must satisfy $(\rho V) = \rho \rho B$. The elements must be integers such that $(1 \leq V) \leq V$ and $V \in 1:V$ (all integers up to the largest element in V but no other values). The rank of B is $1:V$ in 1-origin or $1+1:V$ in 0-origin. The i th dimension of B is $1:(V+1)/\rho B$. The i th coordinate of B becomes the $V[i]$ th coordinate of H . If two or more coordinates of B map into the same coordinate of H , the length along that coordinate is the least of the related dimensions in B .

Special case. If V is a scalar, it is treated as a one-element vector.

Examples.

```
[+M+3 +0'ABCDEF GHIJKL'
ABCD
EFGH
IJKL
```

2 1 \otimes M

```
AEI
BFJ
CGK {R[1;J] B[J;I].}
DHL
```

1 1 \otimes M {R[I] B[I;I]. The diagonal of the matrix.
Note that the length is the shorter
of the two dimensions of the matrix.)

```
[+A3+2 3 4;124
1 2 3 4
5 6 7 8
9 10 11 12
```

```
13 14 15 16
17 18 19 20
21 22 23 24
```

```
2 1 3  $\otimes$  A3
1 2 3 4
13 14 15 16
```

```

5 6 7 8 (R[I;J;K]-B[J;I;K].)
17 18 19 20

9 10 11 12
21 22 23 24

1 1 1 10A3
1 18 (R[I]=B[I;I;I]. The main diagonal.)

1 2 10A3
1 5 9
14 18 22 (R[I;J]=B[I;J;I]. A diagonal slice.)

2 1 20A3 (R[I;J]=B[J;I;J].)

1 14
5 18
9 22

```

The expressions to the right which relate elements of B , J , and R are formed as follows: The indices applied to R are $([I/V], 'JXL...',$ and the indices applied to B are $'JXL...', '[V]$.

BASE VALUE: $R \div A \div B$

The base value function evaluates its right argument as a representation of a number in a general number system described by its left argument. For example, $2\ 2\ 2\ 1\ 1\ 0\ 1$ gives 5; the vector $1\ 0\ 1$ is evaluated as a number represented in base 2. The left argument, $2\ 2\ 2$, contains the radices of the number system. (Radices are ratios between the weightings of the positions.) For the simple case of a vector left argument A , the K th weighting (in 0-origin) is $\times/(-K)+A$. That is, the K th weighting is the product of the last K elements of A . If W is a vector of these weightings, the result for $A \div B$ is $W \div B$. Thus for the case $2\ 2\ 2\ 1\ 1\ 0\ 1$ the result is $+ / 4\ 2\ 1 \times 1\ 0\ 1$.

Requirements for A and B . Except for the special cases below, A and B must satisfy $(1 \div A) \div 1 \div B$ (the last dimension of A must be the same as the first dimension of B). For arrays A and B , the vectors along the last coordinate of A are used to find vectors of weightings, and each vector along the first coordinate of B is evaluated according to each vector of weightings. The weightings are $W \div \times \backslash (A) \div A, 1$. The result is then $W \div B$. The result has as dimensions $(1 \div A), 1 \div B$ (same as the dimensions of $A \div B$).

Special cases. If A or B is a scalar, it is treated as a one-element vector. If the last dimension of A does not match the first dimension of B but one of the two dimensions is 1, that dimension is extended to match the other.

Examples.

```

2+ 60 60 11 2 3
3723      (One hour, two minutes, and
           3 seconds is 3723 seconds.)

0 60 60 11 2 3 (The first element in the
3723      left argument has no effect.)

[A+2 3p2 2 2 10 10 10
 2 2 2
10 10 10

[B+3 4p1 1 3 2 0 1 4 0 1 0 5 3
1 1 3 2
0 1 4 0
1 0 5 3

A1B      (Each vector along the first coordinate of B
 5 6 25 11 is evaluated according to each vector
101 110 345 203 along the last coordinate of A.)

.513 4 5 (Evaluates the polynomial (3*.5+2)+
7.75      (4*.5)+5. The left argument is extended
           to become a 3-element vector.)

```

REPRESENT: $R \leftarrow A1B$

The represent function represents its right argument in the number system described by its left argument. For example, 2 2 2+5 gives 1 0 1. The left argument contains the radices of the number system. For a vector left argument and a scalar right argument, the result is given by the following function:

```

VR←A SREP B;B
[1] T←cA
[2] R←Np0
[3] L1:←(N 0)/0
[4] R[N]+A[N] B
[5] B←B-R[N]
[6] +(B=0)/0
[7] B←B+A[N]
[8] N←N-1
[9] +L1 V

```

This function is a generalization of the usual method of converting between number systems by dividing and finding remainders.

Requirements for A and B. A and B may have any shape. Each element of B is represented according to the radices in each vector along the first coordinate of A. (If A is a scalar, it is treated as a one-element vector for this operation.) The dimensions of the result are $(pA), nB$ (i.e., the same as for outer product).

Examples.

```

      10 10 10T273
2 7 3

      24 60 60T3723 (3723 seconds is 1 hour,
1 2 3              2 minutes, and 3 seconds.)

      2 2 2T31      (High-order information is lost.)
1 1 1

      0 2 2T31      (High-order information is
7 1 1              intercepted by using a zero.)

      10 10 10T34 281
0 2
3 8
4 1

      D→A+3 2p0 10 2 10 2 10
0 10
2 10
2 10

      A1281 323
70 60
2 3

      0 1
      8 2

      1 1
      1 3

```

EXECUTE: $R \leftarrow B$

The execute function performs API statements in the vector or scalar B. A result is returned only if the expression produces a result. When the execute function is performed as the last operation on a line, any result is automatically printed unless specification or a branch was the last operation within the argument. Branching in the argument has no effect, and any statement label is ignored. If execute is applied to a character

argument representing a list, the list is printed and the first list element is returned as the result of the execute function. Note that the present system does not allow expressions exceeding 150 characters to be executed. (Some statements of as few as 87 characters give a *LIMIT ERROR*.)

Examples.

```

      1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 1040 1041 1042 1043 1044 1045 1046 1047 1048 1049 1050 1051 1052 1053 1054 1055 1056 1057 1058 1059 1060 1061 1062 1063 1064 1065 1066 1067 1068 1069 1070 1071 1072 1073 1074 1075 1076 1077 1078 1079 1080 1081 1082 1083 1084 1085 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1096 1097 1098 1099 1100 1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 1116 1117 1118 1119 1120 1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140 1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160 1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180 1181 1182 1183 1184 1185 1186 1187 1188 1189 1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200 1201 1202 1203 1204 1205 1206 1207 1208 1209 1210 1211 1212 1213 1214 1215 1216 1217 1218 1219 1220 1221 1222 1223 1224 1225 1226 1227 1228 1229 1230 1231 1232 1233 1234 1235 1236 1237 1238 1239 1240 1241 1242 1243 1244 1245 1246 1247 1248 1249 1250 1251 1252 1253 1254 1255 1256 1257 1258 1259 1260 1261 1262 1263 1264 1265 1266 1267 1268 1269 1270 1271 1272 1273 1274 1275 1276 1277 1278 1279 1280 1281 1282 1283 1284 1285 1286 1287 1288 1289 1290 1291 1292 1293 1294 1295 1296 1297 1298 1299 1300 1301 1302 1303 1304 1305 1306 1307 1308 1309 1310 1311 1312 1313 1314 1315 1316 1317 1318 1319 1320 1321 1322 1323 1324 1325 1326 1327 1328 1329 1330 1331 1332 1333 1334 1335 1336 1337 1338 1339 1340 1341 1342 1343 1344 1345 1346 1347 1348 1349 1350 1351 1352 1353 1354 1355 1356 1357 1358 1359 1360 1361 1362 1363 1364 1365 1366 1367 1368 1369 1370 1371 1372 1373 1374 1375 1376 1377 1378 1379 1380 1381 1382 1383 1384 1385 1386 1387 1388 1389 1390 1391 1392 1393 1394 1395 1396 1397 1398 1399 1400 1401 1402 1403 1404 1405 1406 1407 1408 1409 1410 1411 1412 1413 1414 1415 1416 1417 1418 1419 1420 1421 1422 1423 1424 1425 1426 1427 1428 1429 1430 1431 1432 1433 1434 1435 1436 1437 1438 1439 1440 1441 1442 1443 1444 1445 1446 1447 1448 1449 1450 1451 1452 1453 1454 1455 1456 1457 1458 1459 1460 1461 1462 1463 1464 1465 1466 1467 1468 1469 1470 1471 1472 1473 1474 1475 1476 1477 1478 1479 1480 1481 1482 1483 1484 1485 1486 1487 1488 1489 1490 1491 1492 1493 1494 1495 1496 1497 1498 1499 1500 1501 1502 1503 1504 1505 1506 1507 1508 1509 1510 1511 1512 1513 1514 1515 1516 1517 1518 1519 1520 1521 1522 1523 1524 1525 1526 1527 1528 1529 1530 1531 1532 1533 1534 1535 1536 1537 1538 1539 1540 1541 1542 1543 1544 1545 1546 1547 1548 1549 1550 1551 1552 1553 1554 1555 1556 1557 1558 1559 1560 1561 1562 1563 1564 1565 1566 1567 1568 1569 1570 1571 1572 1573 1574 1575 1576 1577 1578 1579 1580 1581 1582 1583 1584 1585 1586 1587 1588 1589 1590 1591 1592 1593 1594 1595 1596 1597 1598 1599 1600 1601 1602 1603 1604 1605 1606 1607 1608 1609 1610 1611 1612 1613 1614 1615 1616 1617 1618 1619 1620 1621 1622 1623 1624 1625 1626 1627 1628 1629 1630 1631 1632 1633 1634 1635 1636 1637 1638 1639 1640 1641 1642 1643 1644 1645 1646 1647 1648 1649 1650 1651 1652 1653 1654 1655 1656 1657 1658 1659 1660 1661 1662 1663 1664 1665 1666 1667 1668 1669 1670 1671 1672 1673 1674 1675 1676 1677 1678 1679 1680 1681 1682 1683 1684 1685 1686 1687 1688 1689 1690 1691 1692 1693 1694 1695 1696 1697 1698 1699 1700 1701 1702 1703 1704 1705 1706 1707 1708 1709 1710 1711 1712 1713 1714 1715 1716 1717 1718 1719 1720 1721 1722 1723 1724 1725 1726 1727 1728 1729 1730 1731 1732 1733 1734 1735 1736 1737 1738 1739 1740 1741 1742 1743 1744 1745 1746 1747 1748 1749 1750 1751 1752 1753 1754 1755 1756 1757 1758 1759 1760 1761 1762 1763 1764 1765 1766 1767 1768 1769 1770 1771 1772 1773 1774 1775 1776 1777 1778 1779 1780 1781 1782 1783 1784 1785 1786 1787 1788 1789 1790 1791 1792 1793 1794 1795 1796 1797 1798 1799 1800 1801 1802 1803 1804 1805 1806 1807 1808 1809 1810 1811 1812 1813 1814 1815 1816 1817 1818 1819 1820 1821 1822 1823 1824 1825 1826 1827 1828 1829 1830 1831 1832 1833 1834 1835 1836 1837 1838 1839 1840 1841 1842 1843 1844 1845 1846 1847 1848 1849 1850 1851 1852 1853 1854 1855 1856 1857 1858 1859 1860 1861 1862 1863 1864 1865 1866 1867 1868 1869 1870 1871 1872 1873 1874 1875 1876 1877 1878 1879 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902 1903 1904 1905 1906 1907 1908 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 1931 1932 1933 1934 1935 1936 1937 1938 1939 1940 1941 1942 1943 1944 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085 2086 2087 2088 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100 2101 2102 2103 2104 2105 2106 2107 2108 2109 2110 2111 2112 2113 2114 2115 2116 2117 2118 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2130 2131 2132 2133 2134 2135 2136 2137 2138 2139 2140 2141 2142 2143 2144 2145 2146 2147 2148 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349 2350 2351 2352 2353 2354 2355 2356 2357 2358 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2380 2381 2382 2383 2384 2385 2386 2387 2388 2389 2390 2391 2392 2393 2394 2395 2396 2397 2398 2399 2400 2401 2402 2403 2404 2405 2406 2407 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420 2421 2422 2423 2424 2425 2426 2427 2428 2429 2430 2431 2432 2433 2434 2435 2436 2437 2438 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2460 2461 2462 2463 2464 2465 2466 2467 2468 2469 2470 2471 2472 2473 2474 2475 2476 2477 2478 2479 2480 2481 2482 2483 2484 2485 2486 2487 2488 2489 2490 2491 2492 2493 2494 2495 2496 2497 2498 2499 2500 2501 2502 2503 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526 2527 2528 2529 2530 2531 2532 2533 2534 2535 2536 2537 2538 2539 2540 2541 2542 2543 2544 2545 2546 2547 2548 2549 2550 2551 2552 2553 2554 2555 2556 2557 2558 2559 2560 2561 2562 2563 2564 2565 2566 2567 2568 2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590 2591 2592 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605 2606 2607 2608 2609 2610 2611 2612 2613 2614 2615 2616 2617 2618 2619 2620 2621 2622 2623 2624 2625 2626 2627 2628 2629 2630 2631 2632 2633 2634 2635 2636 2637 2638 2639 2640 2641 2642 2643 2644 2645 2646 2647 2648 2649 2650 2651 2652 2653 2654 2655 2656 2657 2658 2659 2660 2661 2662 2663 
```

```

      v3 4p12
1   2   3   4
5   6   7   8
9  10  11  12

```

```

      p v3 4p12
3   10

```

```

      v'AB'

```

DIADIC FORMAT: $Z \leftarrow V * B$

The dyadic format function represents columns of B according to pairs of integers in the vector V . The first number of a pair in V gives the width, and the second number gives the precision to be used in representing the number. The width is the number of character positions to be used for the column, and if 0 is used, the system chooses a width so that at least one blank will separate that column from the preceding column. The result R has the same dimensions as B except that the last dimension of R is usually greater than the last dimension of B . Character arguments are not allowed. The precision has the following significance:

precision ≥ 0 The numbers are represented in decimal format. The precision is the number of digits beyond the decimal point. If the precision is zero, no decimal point appears.

precision < 0 The numbers are represented exponential format. The number of digits shown is the absolute value of the precision, and if the precision is 1, no decimal point appears. Five columns are required for the exponent, unless the system chooses the width, in which case the number of columns required for the exponent depends on the magnitude of the numbers.

Note that a domain error will result if a number cannot be represented in the space provided. However, there is no requirement that spaces separate numbers in the same row.

Exception cases. If B is a scalar, it is treated as a one-element vector. (Hence the result is a vector, never a scalar.) If V is a scalar or one-element vector, it is extended to become $V \leftarrow 0, V$. (Thus the width of the columns will be chosen by the system.) Then if pV is 2 but $-1 + pB$ (the last dimension of B) is not 1, V is replicated to become $V \leftarrow (2 \times -1 + pB)pV$ so that the pair of numbers in V will be applied to all columns.

Examples.

```
7 34.3456 2.8 926
0.746 2.800928.000

44.3456 2.8 929
0.3456 2.8000 928.0000

1 042 3p1 0

101
010
101

10 -343 1p2.34567 4.23E18 -5.3E-5
2.35E0
-4.23E10
-5.30E-6

-171E5 +.22E6 1.8E2
1E5 1E6 2E2
```

MATRIX INVERSE: $P \leftarrow I \otimes B$

The matrix inverse function returns the inverse of a matrix B . The inverse is such that

$$(BB)^{-1} \cdot B \rightarrow I$$

where I is an identity matrix (i.e., a matrix with 1's along the diagonal and 0's elsewhere) having $1 \times pB$ rows and columns. Note that this uniquely defines B only as long as B has the same number of rows and columns. However, if B has more rows than columns, the result B can be uniquely defined by $B \leftarrow (B(BB)^{-1} \cdot B)^{-1} \cdot B$. The result is related to the result for the dyadic matrix divide function according to $BB \leftarrow I \otimes B$, where I is an identity matrix having $1 \times pB$ rows and columns.

Requirements for B . Ignoring the exceptions below, B must be a matrix such that $(1 \times pB) \geq 1 \times pB$ (B must have at least as many rows as columns) and B must have an inverse. Note that some matrices do not have inverses and produce a *DOMAIN ERROR* if an inverse is requested. In particular, a square matrix with two identical rows or with one row that can be produced by multiplying other rows by factors and adding them has no inverse. Actually, there is no precise distinction between matrices that have inverses and those that do not, and *DCI* is used in the test. Increasing the value of *DCI* may prevent a *DOMAIN ERROR*, but the result so produced is less reliable and may be completely meaningless. The dimensions of the result are $p \times B$ (i.e., $p \times qB$).

Special cases. If B is a scalar, the result is the scalar $+R$. If B is a vector, the result is $B++/B*2$. Except for the result rank, the scalar case is the same as if the scalar were treated as a one-by-one matrix, and the vector case is the same as would be produced by treating the vector as a one-column matrix.

Examples. See the examples at the end of the following discussion of matrix divide.

MATRIX DIVIDE: $R \leftarrow A \div B$

The matrix divide function solves systems of simultaneous equations or finds least-squares regression coefficients. When the matrix B has the same number of rows and columns, R is the solution to linear equations represented by the constant vector A and the coefficient matrix B . When B has more rows or columns, the result R contains the regression coefficients for a dependent variable A and independent variables in the columns of B . Note that the result is the same as $(\#B)+.*A$.

Requirements for A and B . Ignoring the special cases below, B must be a matrix such that $(1+pB) \sim 1+pB$, and B must have an inverse (see the preceding discussion of the matrix inverse function). Also, A must be a matrix such that $(1+pA) \sim 1+pB$ (they must have the same number of rows). When A has more than one column, the result R has a solution column for each column of A . The result has the dimensions $(1+pB), 1+pA$ (one row for each column of B and one column for each column of A). The result R satisfies $B+.*R \leftarrow A$ if B is a square matrix. When B is not a square matrix, the result minimizes each element of

$$+/(A-B+.*R)*2$$

That is, $B+.*R$ gives predicted values for the regression coefficients R , and $A-B+.*R$ gives the residuals; so the sum of the squared residuals is minimized.

Special cases. The arguments may also be scalars or matrices. A scalar is treated as a one-by-one matrix, and a vector is treated as a one-column matrix. After this extension, $1+pA$ must match $1+pB$. The dimensions of the result are $(1+pB), 1+pA$ where A and B here are the original arguments before extension.

Example 1. To solve the system of equations:

$$\begin{aligned} 5 &= x + 2y \\ 4 &= 5x + 3y \end{aligned}$$

Use:

$$[+M+2 \ 2 \ 1 \ 2 \ 5 \ 3$$

$$\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$$

```

      5 4 MW
1 3 (A vector result.)

      (2 105 4) MW
-1
3 (A matrix result.)

```

The answer is $x=-1$, $y=3$.

Example 2. Given $V1=.8 .9 1.0 2.2 3.1$, $V2=1 2 3 1 2$, and $Y=4.5 5.6 9.2 6.3 7.1$ find the values of $A1$ and $A2$ that most nearly satisfy $Y=(A1 \times V1)+A2 \times V2$ in the least squares sense.

```

      Q+Q+V1,(:.5)V2
0.8 1
0.9 2
1 3
2.2 1
3.1 2

```

```

      PPP+3
      Q+Z+Y@Q
1.37 2.56

```

The predicted values for Y are:

```

      Q+.xZ
3.66 5.35 9.04 5.58 9.37

```

and the residuals are:

```

      Y-Q+.xZ
0.845 0.252 3.159 2.72 -2.27

```

Example 3. Using $V1$ and $V2$ from Example 2 and $Y2=5.5 8.6 11.2 10.3 9.1$, find $A1$, $A2$, and $A3$ that most nearly satisfy $Y2=A1+(A2 \times V1)+A3 \times V2$. This problem is like the previous one except that we imagine $A1$ to be the coefficient of a vector of 1's. The solution is given by;

```

      Q+P+1.Q
1 0.8 1
1 0.9 2
1 1 3
1 2.2 1
1 3.1 2

```

```

      Y2@P
5.76 0.593 1.35

```


Chapter 6. Composite Functions

As described in Chapter 2, an operator is a special function that takes functions as arguments and produces a function as a result. Except for the result of the axis operator, these resulting functions are the composite functions. A few examples will help to illustrate this. The expression $+/\mathbf{1\ 2\ 3}$ ("the plus reduction of $\mathbf{1\ 2\ 3}$ ") is the same as $1+2+3$. Similarly, $/\mathbf{1\ 2\ 3}$ is $1-2-3$ or 2 (remember that it is performed from right to left). The function symbol to the left of the slash indicates the particular dyadic scalar function to be used. The forms for composite functions are d/B (reduction), $d\backslash B$ (scan), $A*.dB$ (outer product), and $A d.BB$ (inner product), where d and B represent symbols for any dyadic scalar functions.

REDUCTION: $R=d/[\mathbf{K}]B$

Reduction applies a dyadic scalar function repeatedly between elements in vectors along the K th coordinate of B . For a vector B , the reduction is of the form

$$B[1]dB[2]d \dots dB[B]$$

For higher order arrays the same sort of operation is performed for each vector along the K th coordinate. When the axis operator is omitted the operation is performed along the last coordinate. The alternate symbol $/$ can be used to indicate the operation should be performed along the first coordinate.

Requirements for B . Elements of B must be in the domain of the scalar function used. Thus, character arguments are allowed only for the functions $=$ and $*$. Except for the special cases below, the result has a rank that is one less than the rank of B and the dimensions of the result are $(K*1,cB)/cB$ (the same as the dimensions of B except that the K th dimension of B is missing).

Special cases. A scalar is treated as if it were a one-element vector, and the result is then a scalar. If the length of s along the k th coordinate is 1, the result is the same as the argument except that one dimension is removed. No operation is actually performed in this case, so no check is made to see whether the values are in the domain of the function, except that arguments of character type are still illegal for functions other than = and *. When s is empty but the result is not empty, the result contains the identity element for the function if one exists. The following table shows the identity elements used. Note that in some cases the identity elements are identities in a rather loose sense. Some are right identities only, some are left identities only, some are both, and some are identities only for logical arguments. Functions for which there is no identity in the table produce *DOMAIN* errors when applied along a coordinate of length 0.

Function	Identity	Function	Identity
+	0	*	1
-	0	^	1
x	1	v	0
†	1	!	1
⌈	-1.265322	>	0
⌊		≥	1
=	1.265322	<	0
≠	0	≤	1

Examples.

```

15      ⌈/3 1 9 15      (Largest element.)
1       ⌊/3 1 9 15      (Smallest element.)
120     x/1 2 3 4 5      (Product.)
15      +/1 2 3 4 5      (Sum.)
3       -/1 2 3 4 5      (Alternating sum; same as
                        1+(-2)+3+(-4)+5.)
1.875   +/1 2 3 4 5      (Alternating product; same
                        as (1x3x5)+2x4.)

⌈xP-3 4p12
1  2  3  4
5  6  7  8
9 10 11 12

```

4	8 12	[/P	(Largest element in each row.)
9	10 11 12	[/P	(Largest element in each column.)
3		+/3	(A scalar is treated as a vector.)
5		^/5	(No domain check for one element.)
0		+/10	(An identity if the length is zero.)
0 0 0		+/3 0p0	(An identity for each of the 3 rows.)
		^/AeB	(Gives 1 if all elements of the vector A occur in B.)
		v/AeB	(Gives 1 if any elements of the vector A occur in B.)

SCAN: $R \leftarrow d \backslash [K] B$

Scan performs a series of reductions. For example, $+ \backslash 1 \ 2 \ 3$ 4 5 returns 1 3 6 10 15; that is, the i th element is $+ / i \ B$. For arrays other than vectors, the result has the same shape as the argument, and the elements along the K th coordinate are produced by performing a reduction over the first I elements. Arguments of character type are not allowed. If the axis operator is absent, the last coordinate is assumed. The alternate symbol \downarrow can be used to indicate the operation is to be performed along the first coordinate.

```

      v \ 0 0 1 0 0 1 0
0 0 1 1 1 1 1
      ^ \ 1 1 0 1 0 1
1 1 0 0 0 0
      x \ 1 2 3 4 5 6
1 2 6 24 120 720
      □ • P • 3 4p112
1 2 3 4
5 6 7 8
9 10 11 12

```

```

      +\P
1  3  6 10
5 11 18 25
9 15 30 42

```

```

      +\P      {Same as +\[1]P.)
1  2  3  4
6  8 10 12
15 18 21 24

```

OUTER PRODUCT: $R+A \cdot dB$

Outer product applies a scalar dyadic function using all elements of A as left arguments and all elements of B as right arguments. The rank of the result is $(\rho\rho A)+\rho\rho B$ and the dimensions of the result are $(\rho A), \rho B$. Each result element has as its first $\rho\rho A$ indices the indices of the element used from A and has as its last $\rho\rho B$ indices the indices of the element used from B .

Examples.

```

      1 2 3 *.x4 5 6 7
4  5  6  7
6 10 12 14      (Each element of the left argument is
12 15 18 21      multiplied by each element of the right.)

```

```

      1 2 3 *.z3 1 3
0 1 0
0 3 0
1 0 1

```

```

      +/1 2 3 *. 3 1 3
1 0 2      (The number of 1's, 2's and 3's
            in the right argument.)

```

INNER PRODUCT: $R+Ad \cdot B$

Inner product applies the scalar function D between each vector along the last coordinate of A and each vector along the first coordinate of B , then performs a reduction using d to that result. The usual matrix product is $A+ \cdot B$.

Requirements for A and B . Ignoring the special cases below, the last dimension of A must match the first dimension of B . The dimensions of the result are $(1+\rho A), 1+\rho B$ (all dimensions of A except the last and all dimensions of B except the first).

Special cases. If *A* or *B* is a scalar it is treated as a one-element vector. Then if the last dimension of *A* does not match the first dimension of *B* but one of the two dimensions is 1, that dimension is extended to match the other (thus allowing the array having the 1 as a dimension to be used repeatedly).

Examples. The following table shows examples for arguments of various ranks.

$\frac{ppA}{2}$	$\frac{ppB}{2}$	$\frac{ppR}{2}$	Result
2	2	2	$R(I;J) = d/A[I;]DB[,J]$
2	1	1	$R[I] = d/A[I;]DB$
1	2	1	$R[I] = d/ADB[,J]$

The following examples illustrate useful inner products:

```

      A+2 3p1 0
1 0 1
0 1 0
      B+3 3p1 9
1 2 3
4 5 6
7 8 9
      A+. * B      (Matrix Product.)
8 10 12
4 5 6

      'ABCD'+. = 'XYZCD' (Counts matches in corresponding
2                      positions.)

      C+TABLE+3 4p'FOURFIVRSIX '
FOUR
FIVE
SIX

      TABLE+. = 'FIVE'
0 1 0      (Gives 1 for a row that matches
            'FIVE'.)

```


Chapter 7. System Functions and Variables

This chapter discusses system functions and variables other than `□` and `□` (which were discussed in Chapter 2), other than the ones related to shared variables (which have not been implemented in the present version of APLUM), and other than `□FD` and `□SY`. System functions and variables allow communication with the APL system, and, to some extent, with the KRONOS operating system under the control of which the APL system runs. In most respects system functions and variables behave as other APL functions and variables except that: their names are distinguished by beginning with the symbol `□` or `□`, they control the APL environment in ways that other functions and variables cannot, and the values of system variables can change between settings. For example, `□AI`, which is a vector of accounting information, may be set by the user to any desired value, but the next time he requests its value, it will correctly reflect current accounting information—that is, the system resets the value of `□AI` before it is read. Similarly, `□AI` can be erased by the user, but the system gives it a value whenever its value is requested.

The system variables that affect operation of the APL system have restricted shapes and domains. For example, `□IO`, the origin for indexing, must have a value of 1 or 2. Any attempt to set `□IO` to an improper value will result in a `RANK ERROR` or a `DOMAIN ERROR`. However, the user can erase `□IO` or declare `□IO` to be local to a function and then fail to assign it a value. When a system variable is undefined and its value is required for an operation, an `IMPLICIT ERROR` results. For example:

```

      ?Z←JOTA 3;□IO  (□IO is a local variable.)
[1]  Z←1B 1
      JOTA 3
01: IMPLICIT ERROR
JOTA[1] Z←1B
      /
```

Output Control

- [PP+integer (1 to 15)]**
Printing precision--maximum number of significant digits used for numeric output.
- [PW+integer (33 to $\sim 1+2*17$)]**
Maximum printing width used for output.
- [PL+pagesize,linecount (0 to $\sim 1+2*17$)]**
Print lines. Print lines to be used before a halt to allow the terminal operator to intervene, and count of lines used. If **[PL[1]]** is 0, output will be uninterrupted.

Indicators affecting Primitive Functions

- [CT+number (0 to .01)]**
Comparison tolerance used for relational functions, membership, index-of, and domain tests.
- [IO+0 or 1]**
Index origin. Determines base for counting.
- [RL+integer (1 to $\sim 1+2*47$)]**
Random link used by random number functions.

Function Definition

- [ENV+0 or 1]**
Environment control. Affects **[CR]**, **[PX]**, **[EX]**, **[WC]**, **[NL]**, **[STOP]**, **[TRACE]**, **[LOCK]**, **[LTINE]**, **[NAMESS]**, and **[COEF]**. If **[ENV]** is 0, the global environment is used, and if **[ENV]** is 1, the current environment is used.
- matrix=[CR 'name']**
Canonical representation of a function in the form of a matrix.
- Z=[PX matrix]**
Fixes the function represented by the character matrix argument. The result returned is a vector containing the name of the function, or, if the operation failed, a numeric scalar line number for the erroneous statement.
- vector=[EX 'names']**
Expunges (erases) objects named by the right argument. The result contains 1's for names that are now available, 0's for others.

vector=`[UNC 'names'`

Returns the name class for each name--0 for available, 1 for locked variable (label or group), 2 for unlocked variable, 3 for function, or 4 for distinguished name.

matrix=`[VL V`

matrix=`'letters' [VL V`

The namelist functions return matrices of names in use. Which names are returned depends on class numbers in V--locked variables (labels or groups) if 1;V, unlocked variables if 2;V, functions if 3;V, and defined distinguished names if 4;V. The left argument of the dyadic form should contain letters to further restrict names to those beginning with those letters.

vector=`[LOCK 'names'`

Locks functions and variables named by the right argument. The result is a vector containing 1's for success, 0's for failure.

Stop, Trace, and Timing Control

V `[STOP 'name'`

Sets stop controls for lines specified by V and clears controls for other lines.

Z+ `[STOP 'name'`

Returns a vector of line numbers for which stop controls are selected.

V `[TRACE 'name'`

Sets trace controls for lines specified by V and clears controls for other lines.

Z+ `[TRACE 'name'`

Returns a vector of line numbers for which trace controls are set.

V `[LTIME 'name'`

Selects accumulation of line timing information for lines specified by V and clears time accumulation for other lines. Also clears time totals to 0.

Z+ `[LTIME 'name'`

Returns a matrix with line numbers in column 1 and times (in milliseconds) in column 2.

Program Library Functions

[WSID+ `'name'`

[WSID contains the workspace identification of the active workspace. This name is used when no name is given for [SAVE.

[-]SAVE 'wsname [:passwd][options]'
 Saves a copy of the active workspace under the name specified. **[-]SAVE''** (no name given) uses the name in **WSID**.

A [-]SAVE 'wsname [:passwd][options]'
 Same as above except that **A** controls the state indicator of the active and stored workspaces. If **A** is 0 or 1 the state indicator is cleared or backed up to the last suspension, respectively.

[-]EX+'expression'
 The latent expression is executed immediately after the workspace containing it is loaded.

[-]LOAD '[:account] wsname [:passwd]'
 Activates a copy of a stored workspace and then executes the latent expression if one is defined.

matrix=V [-]NAMES '[:account] wsname [:passwd]'
 Lists names used in a stored workspace. The result is a matrix of names of objects in the name classes specified by elements of **V**--locked variables (labels or groups) if 1cV, unlocked variables if 2cV, functions if 3cV, and distinguished names if 4cV.

matrix=[-]NAMES '[:account] wsname [:passwd]'
 Returns a matrix of all names of classes 1, 2, and 3 in the workspace.

matrix='names' [-]COPY '[:account] wsname [:passwd]'
 Copies specified objects into the active workspace from a stored workspace.

matrix=[-]COPY '[:account] wsname [:passwd]'
 Copies all objects of classes 1, 2, and 3 from the workspace.

[-]DROP '[:account] wsname [:passwd]'
 Removes the stored workspace or file named by the right argument from the indicated library.

Z=[-]LIB '[:account] [name]'
 Returns a matrix containing names, types, and sizes of files in a library. If an account number is given, information is given only for the files that are public or semiprivate or for which the user has access permission. If a name is given, detailed information about that one file is returned.

Error Processing

QTRAP integer

Specifies that errors are to be intercepted by a forced branch to the specified line of the currently executing function.

Z-QERR

QERR is a 3-row matrix of the last error message, the line having the error, and a pointer to the position of the error in the line.

matrix-QSIV vector

The result is a character matrix containing the rows of the state indicator with variables display specified by the right argument. **QSIV** w**QLC** gives the entire display (in either origin).

V-QLC

QLC is a vector of all line numbers appearing on the state indicator.

Miscellaneous System Communication

V-AI

AI is a vector of accounting information. **AI** [1 2 3 4] gives: an encoding of the user's account number, accumulated central processor time, accumulated connect time, and accumulated keying time.

V-AI

Atomic vector of all 256 APL characters.

V-TTS

Time stamp: current year, month, day, hour, minute, second, and millisecond.

V-TT

Terminal type.

QWA+V

Working area: **QWA**[1] is the part of the maximum field length available for use, **QWA**[2] is the current field length, **QWA**[3] and **QWA**[4] are the minimum and maximum field lengths the user wishes APL to use.

QTN 'command'

Terminal mode: commands are **SYSTEM**, **OFF** and **ABORT**, to return to KRONOS command processor, sign off, or abort batch job.

S-QL seconds

Cause execution to delay for the specified number of seconds.

However, three system variables are so important that when they are undefined the system uses default values. Thus, when `PP` is undefined the system uses 30 as the printing width. When `PP` is undefined, normal output uses a value of 1. When `CT` is undefined, the system uses zero as the comparison tolerance for domain tests, although numerical comparisons still give implicit errors. For example,

```

      DEX'CT'
1
      3 3
01: IMPLICIT ERROR (Because CT is undefined for comparison.)
      3=3
      /
      13
1 2 3
      13+1E~12
03: DOMAIN ERROR (Because CT is zero for domain tests.)
      13+1E~12
      /

```

Certain system variables are not stored in the workspace. These session variables remain in effect if another workspace is loaded and always have their normal values when an APL session begins. The session variables are `NA`, `PL`, `TT`, `DS`, and `AI`.

NAME LISTS

Some system functions require arguments consisting of lists of names. In all cases such name lists can be either a vector of names separated by spaces, or a matrix of names with one name in each row. In either form extra spaces are allowed before or after names. When a system function returns a list of names as a result, the list is always in the form of a matrix because the matrix form is usually more convenient for manipulation by the program.

WORKSPACES

An APL workspace comprises variables, user-defined functions, the state indicator, and system variables that are currently defined. A clear workspace comprises the following:

```

an empty state indicator
PP+10 (printing precision of 10 digits)
PW+120 (up to 120 characters are printed per line)
CT+5E~11 (comparison tolerance is 5E~11)
IO+1 (index origin is 1)
RL+16807 (random link is 16807)
ENV+1 (local environment)
ERR+3 0p''

```

In addition, the clear workspace also presently includes the *SP* functions *DEF* (which performs function definition mode), *END* (which performs system commands), and the mixed functions *Q*, *A*, *r*, and *i*. Although some of these may eventually become incorporated into the APL system itself (and thus be removed from the clear workspace), they presently occupy considerable space in active and stored workspaces. See Chapter 11 to learn how to eliminate or restore these functions.

As functions and variables are defined, they become part of the active workspace. A copy of an active workspace can be saved. To use it at a later time, a copy of the saved workspace can be activated (that is, made active).

A stored workspace is a special kind of KRONOS "file." Under an account number (or user number) can be stored as many files as are allowed by restrictions imposed on the account number. The collection of files is known as a library.

APL workspaces and data files are ordinarily KRONOS *private* files, which means that other users cannot use them. A user may optionally save a workspace as a *semiprivate* file or *public* file by use of commands of the form *[SAVE 'name/S' or [SAVE' name/PU'*. This allows other users to access the workspace but does not allow them to alter it. Other users can be given permission to access a private file by use of the KRONOS PERMIT control card (see Chapter 12). This gives selected user numbers permission to access the particular file. Further details about these file categories can be found in Chapter 9 and Chapter 12.

Passwords can be given to workspaces for additional security. When a workspace is given a password, other users must provide the password in order to access the workspace. However, the owner of the workspace need not provide the password in order to use it.

The first time a workspace is saved it can be given a password or a category (i.e., *private*, *semiprivate*, or *public*). Thereafter, the file password and category remain unchanged for subsequent save commands that replace the stored workspace. (Thus, the password and category options should not be provided for subsequent save commands.) To change the password or category you must load the workspace, drop the stored one, and then resave it with the new options. Alternatively, you can use the *CHANGE* control card (see Chapter 12).

Workspaces can optionally be saved in *direct access* form (ordinarily they are saved in *indirect access* form). This option is chosen by using a command of the form *[SAVE' name/DA'* the first time the stored file is established. *Direct access* workspaces are faster to load, save, or copy, but require more disk space.

The *direct access* option is appropriate for unusually large workspaces that are loaded or saved very often. A workspace can be changed to *direct access* form by loading it, dropping it, then resaving it using the *EA* option.

Workspace names must begin with a letter, which may be followed by additional letters and digits. However, the name must not exceed seven characters. Passwords may consist of 1 to 7 letters or digits.

NOTATION

Throughout this chapter, brackets are used to surround optional portions of expressions. The brackets themselves should not be used. For example,

```
[LOAD '[*account] wsnam [ :passwd ]'
```

means that the account number and password are optional. Any of the following commands are of the correct form:

```
[LOAD 'ALGEBRA'
[LOAD '*A123456 ALGEBRA:SESAME'
[LOAD 'ALGEBRA:SESAME'
[LOAD '*A123456 ALGEBRA'
```

SYSTEM VARIABLES FOR OUTPUT CONTROL

Printing precision. [PP=integer (2 to 15)]

The value of [PP] determines the maximum number of significant digits to be used for numeric output. The result is rounded to [PP] digits; hence if [PP] is 3, 0.34567 would be printed as 0.346. See Appendix B for further details of numeric output format.

Printing width. [PW=integer (30 to ~1+2*17)]

The value of [PW] determines the line width used for output. When a line of output requires more character positions than [PW], the remaining characters are indented and continued on successive lines. Output of numbers will not cause individual numbers to be split between two lines, but output of character data representing numbers may cause numbers to be split between lines.

Print lines. [PL=pagenize, linescount]

[PL] is primarily intended to facilitate the use of CRT terminals having a screen smaller than the total amount of output generated. Appropriate setting of [PL] causes output to pause when the screen has been filled to allow the screen to be examined or cleared (if required) before more output is sent.

The first element of `[PL` should be set to the number of lines that will be used for actual output. The second element of `[PL` is a count of the number of lines actually used for input and output. When each output line or input line has been completed, `[PL[2]` is incremented by 1. If `[PL[1]=[PL[2]`, the system prints ? on the next line and suspends further output until RETURN is pressed. (Any other input is treated as if RETURN has been pressed.) The program requesting input can be halted by use of an interrupt (see Appendix C). When RETURN is pressed, `[PL[2]` is reset to 0, and further output is sent. The value of `[PL[2]` can be reset to compensate for screen repositioning caused by graph mode output. The elements of `[PL` are restricted to positive integer values. If an attempt is made to set `[PL[1]` to 1, it actually is set to 0. If the last line on the screen is used for input, the ? is suppressed and normal input can be entered on that line. (The input request gives a pause to allow the screen to be read.)

`[PL` has a different meaning when APL output is sent to a file rather than to a terminal. Specifically, if APL is not being used from a terminal or is being used from a terminal but the output file name is not OUTPUT, and if the shifted output option is in effect (see Appendix D), a page eject carriage control character is sent at the beginning of the next output line whenever the page size has been exhausted.

VARIABLES AFFECTING PRIMITIVE FUNCTIONS

Comparison tolerance. `[CT=number (0 to .01)`

The comparison tolerance is used when comparing numeric values and when testing whether values are sufficiently close to integers:

1. Two numbers *A* and *B* are considered equal only if

$$(|A-B| \leq [CT \times B$$

2. A number *B* is considered to be in the integer domain if

$$(|(NINT B) - B| \leq [CT + [CT \times NINT B$$

where *NINT B* is the nearest integer to *B*, defined by:

$$[1] \quad Z = (NINT B) \\ [1] \quad Z = (B) \times [.5 + |B|]$$

The value actually used for the operation is *NINT B*. If `[CT` is undefined, zero is used as `[CT`.

Random link. $URL + integer (1 \text{ to } 1+2+47)$

URL determines the next random number to be produced by roll or deal. Each time a random number is requested, the value of URL changes. A series of random numbers can be recreated by setting URL to the same initial value and repeating the same requests. Because the value of URL is saved with the workspace, it may be desirable to reset it after the workspace is loaded to a value based on the current time of day so that the random numbers produced will not be the same as for the last session; for example, $URL++/CTS$.

Index origin. $[IO+0 \text{ or } 1]$

The index origin determines the origin for counting coordinates or elements along coordinates. In 0-origin the elements of a vector would be numbered 0, 1, 2, etc. All indexing should use values that are 1 less in 0-origin than in 1-origin. In addition, the following functions produce results that are 1 less in 0-origin than in 1-origin: $A \uparrow B$, $\uparrow B$, $\downarrow B$, $A \uparrow B$, and $\uparrow B$. In addition, the left argument for dyadic transpose should be 1 less for 0-origin, and all axis operators require values that are 1 less. That is, k should be 1 less in expressions like $A \uparrow k \downarrow B$ and $\phi[k]B$.

FUNCTION DEFINITION

Environment. $[ENV+0 \text{ or } 1]$

$[ENV]$ controls whether the functions $[CR]$, $[FX]$, $[EX]$, $[NC]$, $[NL]$, $[LTIME]$, $[NAMES]$, $[COPY]$, $[STOP]$, $[LOCK]$, and $[TRACE]$ refer to the global environment or to the current environment. When $[ENV]$ is 0, the global environment is used, and when $[ENV]$ is 1, the current environment is used. The normal value of $[ENV]$ is 1, so the system functions listed above may refer to local variables and functions. However, when function definition mode is entered or when a system command is performed, only the global functions and variables are used. When the state indicator is empty, the current environment and the global environment are the same and $[ENV]$ has no effect.

Canonical representation. $matrix + [CR 'name']$

Canonical representation returns a character matrix representation of a function. The right argument contains a character vector or scalar containing the name of the function to be returned. The result will have one row for each line of the function, including the function header. Lines will be indented one space unless they have labels or begin with a comment. If the argument does not name a function in the environment specified by $[ENV]$, a *NAME NOT FOUND* error is given. If the function named by the argument is locked, the result will have 0 0 as its shape.

Fix. $2 + [FX] matrix$

$[FX]$ establishes the function represented by the character matrix argument. If the attempt to establish the function is

successful, Z will be a vector containing the name of the function. Replacement of previously existing functions is allowed and may result in *SI DAMAGE* if the function is halted. The *SI DAMAGE* error is processed as a normal error, except that if the state indicator entry for the currently executing function was damaged, error trapping is not allowed to take place. In this case the error is considered to be located at the last line entered in immediate execution mode. `□FX` cannot be used to replace objects other than functions. An attempt to establish a function may also fail as a result of an incorrectly formed function header or duplicate use of statement labels or local variables. If the attempt fails, Z will contain a scalar row index of the line that was improper. Functions created by `□FX` can be declared local to other functions.

Expunge. `Z+□EX 'names'`

`□EX` expunges (erases) functions and variables named by the argument. The result Z is a logic vector containing 1's in positions corresponding to names in the argument that are now free, and 0's in positions corresponding to names that remain unavailable for new uses. Erasure of a function that is on the state indicator does not take effect until the function is no longer on the state indicator. Thus a function can erase itself and not actually be erased until it exits. The unfinished execution can complete, but the name is immediately available for new uses.

Name class. `vector+□NC 'names'`

`□NC` returns information about use of the names in the right argument. The result Z contains 0, 1, 2, 3, or 4 according to whether the name is available (not in use), a locked variable (label or group), unlocked variable, a function, or other (i.e., a defined distinguished name), respectively. Incorrectly formed names in the argument cause a *DOMAIN ERROR*.

Name list. `matrix+□NL Y or Z+'letters' □NL ↓`

The name list functions return lists of names in use. The right argument is a numeric vector such that `*/Vel 2 3 4`. Y indicates the classes of names for which information is desired--1 for locked variables (labels or groups), 2 for unlocked variables, 3 for functions, and 4 for distinguished names. The result Z is a matrix of the names. The left argument of the dyadic form may contain any number of letters, and names appear in the result only if they begin with those letters.

Lock. `vector+□LOCK 'names'`

The variables and functions specified by the right argument are locked. A locked function cannot be displayed, and a locked variable cannot be reset using specification. (However, a locked variable can be reset by erasing it and then using specification.) Locking a variable is a very useful way to find where the variable is reset. When the variable has been locked, the next assignment to it will cause an error halt. Label

variables and groups are automatically locked to prevent them from having improper values. The result returned by `LOCK` contains 1's in positions corresponding to names that are now locked and contains 0's for other names.

STOP, TRACE, AND TIMING CONTROL

The functions `STOP`, `TRACE`, and `ELTIME` are closely related. In each case the right argument is a character vector or scalar that names a function, and the left argument for the dyadic form must contain nonnegative line numbers for which the control is to be set. Setting controls for any lines clears all controls of the same type for the other lines of the function. Elements of the left argument not in the range of line numbers are ignored. In all cases, an empty vector of line numbers can be used to clear the controls. An empty character vector is allowed as a left argument for notational convenience (e.g., '`STOP PLOT`'). The monadic forms of the functions return information about controls that are currently set.

Stop control. `V STOP 'name'` and `Z+STOP 'name'`

When the stop control is set for a particular line, execution of the function suspends before execution of the line begins, and the system prints `STOP SET`, the name of the function, and the line number. To continue execution where it stopped, issue a branch to the line number just printed. Stop control at line 0 of a function causes suspension just prior to exit from the function. The monadic form returns a vector of line numbers for which stop controls are currently set.

Trace control. `V TRACE 'name'` and `Z+TRACE 'name'`

Setting trace control for a line causes the function name and line number to be printed each time after the line has been executed, and if the result of the line was used for a branch or assignment, the result is printed even though it ordinarily would not be. Setting trace control for line 0 causes tracing of the exit from the function and causes printing of the explicit result of the function (if it has one). The monadic form returns a vector of line numbers for which trace controls are set.

Line timing control. `V ETIME 'name'` and `Z+ETIME 'name'`

Setting the line timing control for a line causes the Central Processor time for that line to be accumulated. The time for a line is accumulated until line timing controls for the function are reset, at which time all accumulated times are set to zero. An attempt to set line timing control for line 0 of a function causes a `DOMAIN ERROR`. The result returned by the monadic form is a 2-column matrix--the first column contains the line numbers for which the line timing control is set, and the second column contains the total times for the lines. Because the time clock has a resolution of one millisecond, each parcel of time used by the line is measured with limited accuracy, and lines consuming

very little time or lines consuming time in small parcels can be expected to show relatively large inaccuracy in accumulate times. Note that the times accumulated for a recursive function can count the time more than once.

PROGRAM LIBRARIES

Workspace identification. `[WSID+name]`

The variable `[WSID]` contains the name of the active workspace. The name of the active workspace is used as the name for storing the workspace if no name is specified when `[SAVE]` or `)SAVE` is used. The name must begin with a letter, which may be followed by additional letters or numbers. No spaces are allowed within the name, but spaces may precede or follow the name. The name must not exceed seven characters.

Save. `2+SAVE 'wsname [:passwd]/[options]`

`[SAVE]` saves a copy of the active workspace under the specified name and attaches to the saved workspace the password if one is used. If a password is used, it must be separated from the name by a colon. The name itself may be omitted, and in this case the value of `[WSID]` is used as the name. When `[SAVE]` is executed from a function, the state indicator of the saved workspace will show suspension where `[SAVE]` was executed. The options may include `S` for `public`, or `PU` for `semiprivate`, `private`, or `public` category) or may include `DA` or `IA` for direct access or indirect access. The list of options may include any desired number of options, separated by spaces, as long as the options do not include contradictory choices. The options and password may be specified only when the saved workspace is first established. If no options are specified, the workspace is saved as a `KRONOS` indirect access private file if the saved workspace is being created; otherwise it is saved in the same form as before.

The result returned is a vector of the workspace name and the current date and time. However, when `[SAVE]` is used in immediate execution mode, the name, date, and time are printed rather than being returned as a result.

Dyadic save. `A [SAVE 'wsname [:passwd]/[options]`

The dyadic save function is like the monadic form except that it permits control over the state indicator in both the active and the saved workspace. The argument `A` may be a numeric scalar or vector. If `A` is 0, a clear state indicator results, and if `A` is 1, the state indicator is backed up to the point of the most recent suspension (or cleared if there have been no previous suspensions). Note that a function calling the dyadic `[SAVE]` function always ceases to execute because of the change in the state indicator, unless an error prevented completion of the operation. Dyadic save prints the workspace name and the current date and time.

Latent expression. `[LX+ 'expression']`

The latent expression in a workspace is executed immediately when the workspace containing it is loaded. When a workspace has no latent expression, the keyboard unlocks for the user to specify the first operation to be performed. A successful load operation ordinarily causes the time and date when the workspace was saved to be printed, but when the workspace contains a latent expression this message is absent.

Load `[LOAD '[*account] wname [:passwd]'`

The function `[LOAD` activates a copy of a stored workspace. The right argument must contain the name of the workspace to be loaded, the password for the workspace (if it requires one), and the account number under which the workspace is stored (if different from the user's own). A successful load results in execution of the latent expression if the workspace being loaded has one. If the workspace has no latent expression, the time and date when the workspace was saved are printed.

Name list for stored workspaces. `matrix+V [NAMES '[*account] wname [:passwd]'`

The `[NAMES` function returns a matrix list of the names used in a stored workspace. The list returned is controlled by `[ENV` in the active workspace. The right argument is the same as the right argument for `[LOAD`. The vector `V` may contain the integers 1, 2, 3, or 4 to specify what classes of names should be returned--locked variables (labels or groups) if 1<`V`, unlocked variables if 2<`V`, functions if 3<`V`, and distinguished names if 4<`V`.

Monadic name list. `matrix+[NAMES '[*account] wname [:passwd]'`
Returns a matrix of names of all objects in the workspace or a vector error message. Same as dyadic form with 1 2 3 as a left argument.

Copy. `matrix+ 'names' [COPY '[*account] wname [:passwd]'`

The function `[COPY` copies functions and variables from a stored workspace to the active workspace. The account number, workspace name, and password are the same as described for `[LOAD`. The list of names in the left argument specifies objects to be copied. However, if copying the object would cause replacement of objects already in the active workspace, the copying process is inhibited. If `[ENV` is zero, copying will be from the global environment of the stored workspace to the global environment of the active workspace, and if `[ENV` is 1, the current environments will be used. The result from `[COPY` is a matrix of names of objects not copied because they were not found, because `WS FULL` occurred, or because they already were in use in the active workspace.

Monadic copy. `error+[COPY '[*account] wname [passwd]'`

Like dyadic copy except that all objects of classes 1, 2, and 3 (see `[WC`) are copied.

Drop. error←[DROP '[*account] wsname [:passwd]'

The function [DROP removes a stored workspace (or other KRONOS file) from the user's library. A password must be specified if an account number is specified and differs from the one used to sign on to the system and if the file has a password.

Library list. list←[LIB '[*account] [name]'

The function [LIB returns the names of files stored under the specified account number (or the user's own account number if no account number is specified). The list is a matrix such that each row has the following fields:

File name: 7 characters
File type: 2 characters
File size (in words): 6 characters

One space separates adjacent fields. When a file name is given, detailed information about that particular file is returned. The format when a name is provided is illustrated below:

```
      [LIB '*APL1 FILESYS'  
FILESYS WS  2059  
IA S  RD  
75/05/12 11:45:58      (When created.)  
75/05/30 13:03:30      (Last change.)  
75/07/31 12:30:59      (Last access.)
```

The first row gives the name, type of file (WS for workspace, F for APL file, blank for all others), and the size in words. The second row indicates the file is indirect access (the other possibility would be DA for direct access), the file category (S for semiprivate, P for private, and PU for public), and the mode of access permitted for other users, (RD for read, WR for write, RM for read-modify, MD for modify, AP for append, RA for read-append).

ERROR PROCESSING

Some system functions respond to certain error conditions by returning a result to indicate the error. However, APL handles most errors by suspending execution at the point of the error, printing a message, and unlocking the keyboard for a new command to be entered. Note that a keyboard interrupt (see Appendix C) is treated as an error, as is typing @ (0, BACKSPACE, 1, BACKSPACE, 7). However, halts due to stop controls are not errors. Special exceptions arise when the error is in an argument to the execute function, in a quad input entry, in a locked function, or when [TRAP has been used to intercept errors.

Errors in an argument to the execute function normally cause two error messages to be printed. The first shows the execute argument, and the second shows the error at the line where execute was called (more precisely, the most recent pendent line other than lines of locked functions or arguments to execute).

Errors in lines entered for quad input cause the request for input to be repeated. If the error was encountered in a function called by the input line, the request for input is not repeated and normal error processing ensues.

For security reasons, lines of a locked function are not shown in error messages. Any error in a line of a locked function is treated as if it were situated in the line where the locked function was called (more precisely, the most recent pendent line other than lines of locked functions or arguments to execute).

The function `QTRAP` can be used to designate a line of the currently executing function to intercept errors. Once this has been done, error trapping is in effect and an error in any line of the function causes a forced branch to the trap line. The error trap is in effect for functions called by that function or for functions that are in turn called by those it calls, etc.

The scope of error trapping is analogous to the scope of local variables. A function with a trap line remains in control of errors unless a function called by it sets its own trap line. The newer trap line takes precedence over the old one until the called function completes execution or clears its trap. The trap also takes precedence over the normal processing of errors in quad input lines.

When a workspace is loaded, an interrupt may be acted upon as an error before the latent expression has been executed and the error trap has been enabled. To prevent this situation, a function with a trap can be halted using a stop control before the workspace is saved. The latent expression can then deliberately cause an error in order to invoke the trap line.

Error matrix. QERR

The character matrix `QERR` contains the last error message. Row 1 has the type of error. Row 2 has the name of the function (truncated to 8 characters for long names) the line number (surrounded by brackets), and the line itself. Row 3 of `QERR` has a slash to indicate where the error was found in row 2. The number of columns in `QERR` varies according to the longest of the three rows.

The first row always shows the type of error actually encountered, but the location of the error as shown in rows 2 and 3 can be different from the actual location of the error under the following conditions:

1. If error trapping is in effect, the error is treated as an error in the pendent line of the trapping function.

2. If error trapping is not in effect and the error occurred in a line of a locked function or in an argument to execute, the location of the error is considered to be the most recent pendent line that is not an argument to execute or a line of a locked function. However, an error in a locked function that uses trapping causes `ERR` to contain a line of the locked function. It is advisable for the locked function to localize `ERR` in order to protect its security.

Trap set. `TRAP` integer

The `TRAP` function sets, resets, or clears the trap line for the currently executing function. Use of `TRAP` from immediate execution mode has no effect. The argument must be an integer. If the integer is within the range of line numbers, that line becomes the trap line. If the number is 0 or exceeds the number of lines, trapping causes exit from the function. The trap can be cleared by `TRAP 0`. Once trapping is in effect, an error in that function, in `input`, or any function invoked by it causes a forced branch to be taken to the trap line, and the trap state is cleared. Note that `TRAP` must be used to set the trap again before additional errors can be intercepted by that function. Hence a second error during processing of the trap routine results in either normal error processing or error processing by a function that invoked this one. If trapping is in effect, execution of functions can still halt as a result of a stop control. However, the trap then remains in effect for errors in immediate execution mode.

When a forced branch to the trap line occurs, at least one function will execute before an interrupt is detected. For complete security, the trap line can immediately reset the trap.

Location counter: `LOC`

The variable `LOC` contains a vector of all line numbers appearing on the state indicator. The numbers appear in the same order as in the `JSIV` display--that is, the numbers of the most recently invoked lines appear first. The first element is the number of the function line currently executing.

State indicator and variables. `matrix=JSIV` vector

The function `JSIV` returns rows of the state indicator, including local variables. The argument must be a vector or scalar containing integers. The value returned is a character matrix containing a portion of the `JSIV` display selected by the right argument. `JSIV upLOC` prints the entire `SIV` display (in either origin). If a value in the argument exceeds the range of appropriate row indexes for the `JSIV` display, a blank line

appears in the corresponding row of the result. Note that only entries for function lines appear on the state indicator -not execute arguments, quad input lines, or immediate execution lines.

MISCELLANEOUS SYSTEM COMMUNICATION

Accounting information. $\square AI$

The variable $\square AI$ is a numeric vector of the following accounting information:

- $\square AI[1]$ - A numeric encoding of the user's account number. For a character vector V containing the 7-character account number, the value of $\square AI[1]$ is generated in one origin by 1001:'ABCDEFGHIJKLMNORPQRSTUUVWXY20123456789'. $\backslash V$
- $\square AI[2]$ - Central processor time used.
- $\square AI[3]$ - Total connect time.
- $\square AI[4]$ - Total time the keyboard has been unlocked. Includes part of the time required for the system's response.

Times are in milliseconds and are cumulative since signing on to APL.

Atomic vector. $\square AV$

The vector $\square AV$ contains all 256 characters manipulable by APL. The ordering is such that the first 128 characters are in ASCII order. Note that the ordering of characters in $\square AV$ is system dependent, and programs that depend on the ordering of characters in $\square AV$ cannot be easily transferred to other APL systems. See the table in Appendix C to find positions of particular characters.

Time stamp. $\square TS$

The value of $\square TS$ is a 7-element numeric vector expressing the current point in time. The elements are in the following order: the year (e.g., 1975), month (1 for January), day of the month, hour (0 to 23), minute, second, and millisecond. The last element is always 0 because the operating system does not report the time of day to millisecond precision.

Terminal Type. $\square TT$

The value of $\square TT$ identifies the type of terminal in use. The value is a numeric scalar as follows:

- 1 - Correspondence
- 2 - Type-pairing
- 3 - Bit-pairing
- 4 - ASCII-APL
- 5 - Teletype Model 33

- 6 - Full ASCII
- 7 - Batch ASCII
- 8 - Batch 501 Printer
- 9 - Teletype 38, arrangement 3

Working area. `[WA]`

The value of `[WA]` is a 4-element vector of: the part of the maximum field length available, the current field length, the minimum field length the user wishes used, and the maximum field length the user wishes used. The field length is the actual memory space occupied by the APLUM system and the workspace. The user can set constraints on the field length to be used in order to optimize performance (see Chapter 11). Attempts to reset the first two elements of `[WA]` have no effect. The maximum field length cannot be set to less than that which is currently required. Setting `[WA][4]` to more than the user's validation limit (see Chapter 12) or more than the field length limit imposed by the computer operator may actually increase the chance of a `WS FULL` error.

Terminal mode. `[TM]command`

The terminal mode function allows the following operations:

- `[TM]SYSTEM` Returns control to the KRONOS command processor.
- `[TM]OFF` Signs the user off.
- `[TM]ABORT` Terminates job with KRONOS abort error flag set.

Note that these commands do not cause the active workspace to be saved.

Delay. `3+[DL]seconds`

Causes execution to delay for the number of seconds requested. The delay does not involve consumption of Central Processor time. The result returned is the actual delay that occurred (usually slightly more than requested). The delay cannot be interrupted.

Chapter 8. System Commands

System commands in APLUM provide the same capabilities as some of the system functions and variables. The system commands are provided for compatibility with other APL systems. There are some advantages to using system functions and variables instead of system commands--the system functions and variables can be used in programs (system commands cannot) and system functions and variables are more efficient. For more complete discussions of the operations performed by system commands, see the related system functions in Chapter 7.

GROUPS

The APLUM system, unlike some other APL systems, does not have a distinct data type for "groups." However, the APLUM system commands allow a character matrix of names to be used for the same purposes as groups in the other systems. For example, if *GRPX* is a matrix of names, the command `)ERASE .GRPX` would erase *GRPX* and any objects referenced by the names in *GRPX*. The period in the command is required to indicate that objects referenced by *GRPX* are to be erased, not just *GRPX* itself. The general system convention for distinguishing groups is that all group names should begin with *GRP*. Matrices of names that do not begin with *GRP* can be used as groups, but they will not be listed by the command `)GRPS`. The names in the group definition can be preceded by a period, which causes them to be interpreted as a reference to another group.

`)CLEAR` (Equivalent to `!LOAD'APLO CLEARWS'`)

The command `)CLEAR` activates a clear active workspace (described in Chapter 7) and erases all indirect access files and unties all direct access files.

```

)CLEAR
    Activates a clear workspace.
)ERASE names
    Erases specified functions and variables.
)SAVE [wname] [:passwd] [/options]
    Saves a permanent copy of the active workspace.
)LOAD [*account] wname [:passwd]
    Activates a copy of the specified workspace.
)DROP [*account] wname [:passwd]
    Removes a permanent workspace from the library.
)COPY [*account] wname [:passwd] [names]
    Protected copy of all global objects of classes 1, 2, and 3
    or selected global objects from a stored workspace to the
    active workspace.
)UCOPY [*account] wname [:passwd] [names]
    Unprotected copy of all global objects of classes 1, 2, and
    3 or selected global objects from a stored workspace to the
    active workspace.
)LIB [*account] [name]
    Displays names, types, and sizes of all files, or displays
    detailed information about a single file.
)SYSTEM
    Returns control to KRONOS command processor.
)OFF
    Signs a user off.
)SI
    Displays the state indicator.
)SIV
    Displays the state indicator along with names of variables.
)FNS [letter]
    Displays names of functions.
)VARS [letter]
    Displays names of variables.
)GRPS [letter]
    Displays names of groups.
)GRP group-name
    Displays names in a specified group.
)GROUP group-name names
    Forms a group having specified names.

```

ERASE *names* {Equivalent to `^X 'names'` }

Erases all global objects specified by the list of *names*. If a name is preceded by a period, the name is treated as the name of a group. The erasure erases the group itself (actually a matrix of characters) and the objects referenced by the group.

SAVE [*wsname*] [:*passwd*] [/options]
(Equivalent to `USAVE '[:passwd] [/options]'`)

The **SAVE** command saves a copy of the active workspace under the name specified or under the name in `WSID` if no name is given.

LOAD [**account*] *wsname* [:*passwd*]
(Equivalent to `LOAD '[:passwd] wsname [:passwd]'`)

The **LOAD** command activates a copy of a stored workspace. A password is required if the workspace has a password and is stored under another user number. After the workspace has been loaded, the system executes `DLX` if `DLX` is defined.

DROP [**account*] *wsname* [:*passwd*]
(Equivalent to `UDROP '[:passwd] wsname [:passwd]'`)

The **DROP** command removes a stored workspace or other XROMS file from a library. If the workspace is in another user's library, a matching password must be given if the stored workspace has a password. The user must also be authorized to alter the existing file.

COPY [**account*] *wsname* [:*passwd*] [*names*]
UCOPY [**account*] *wsname* [:*passwd*] [*names*]

The **COPY** command performs a protected copy of global functions and variables from a stored workspace to the active workspace. The **COPY** command will not replace objects in the active workspace with objects from the stored workspace having the same names. The **UCOPY** command performs an unprotected copy and will replace objects having the same names. If no list of names is given, all objects of classes 1, 2, and 3 are copied. If a name in the list is preceded by a period, the name is assumed to refer to a group and objects named in the group are also copied. The `UCOPY` function can be used instead of **COPY** if groups are not to be copied. The form is `['names'] UCOPY '[:passwd]'`.

)LIB [+account. lname. (Equivalent to **[LIB '[*account]lname']**)

The **)LIB** command displays names, types, and sizes of all files the user is authorized to access, or, if a file name is specified, **)LIB** displays detailed information about that particular file. The format is the same as for **[LIB** (see Chapter 7).

)SYSTEM (Equivalent to **[TN'SYSTEM'**)

The command **)SYSTEM** causes the user to leave APL control and allows the KRONOS command processor to execute subsequent commands. The active workspace is not saved.

)OFF (Equivalent to **[TN'OFF'**)

The **)OFF** command signs a user off the system.

)SI

)SIV (Equivalent to **[SIV 1p[LC]**)

The command **)SI** lists the state indicator, and the command **)SIV** lists the state indicator and all local variables. See Chapter 2 for the format of the display.

)FWS [letter] (Equivalent to **[NL 3]**)

)VARS [letter] (Equivalent to **[NL 2]**)

)GRPS [letter] (Roughly equivalent to **'G' [NL 1]**)

These commands list the names of defined global functions, variables, and groups, respectively. If a letter is included, only names beginning with that letter or letters that follow that letter in the alphabet are shown. The command **)GRPS** lists variable names that begin with **GRP**.

)GROUP group-name names

The command **)GROUP** defines a group, extends a group, or erases a group definition. Groups are actually represented as character matrices. If the group-name itself is the first name in the list of names, any previously defined group is extended by the addition of the remaining names. If no names are given, the group definition is erased but objects named by the group are not erased. Names listed in the command can be preceded by a period in order to include a period in the group definition (to indicate the name refers to another group).

`)GRF grprame` (Equivalent to `grprame`)

The command `)GRP` displays the definition of the indicated group. If the group is not defined or is not a character matrix, an error message is given.

Chapter 9. File System

This chapter discusses files from the APL user's point of view. The APLUM system supports two distinct types of files: APLUM files, and KRONOS coded files. The use of files enables programs to deal with large quantities of data that would not fit into a workspace, and files also provide a convenient way for programs to communicate with one another.

Further information about KRONOS files can be found in the KRONOS 2.1 Reference Manual.

APLUM FILE CONCEPTS

An APLUM file is a collection of APL arrays with each array identified by a nonnegative integer. The following example shows creation of a file and writing and reading a few records (arrays) of the file.

```
)LOAD *APL1 FILESYS (File system functions are
                      loaded from APL1.)

'SAMPLE' FCREATE 9 (The FCREATE function is used
                   to create a file with the name SAMPLE
                   and with 9 as its number.)

'RECORD 3' FWRITE 9 3 (The left argument is written
                      to file 9 as record 3.)

(3 3p19)FWRITE 9 1

(2 3p'[]') FWRITE 9 28

PREAD 3 1 (The records can be read in
1 2 3      any order.)
4 5 6
7 8 9
```

Summary of File Functions.

[*account] filename [password] [/options] FCREATE fnum
Creates a file. Options are DA, C, WR, S, or PU.

array FWRITE fnum[,rnum]
Writes array on file number fnum as record rnum.

result=FREAD fnum[,rnum]
Reads the record numbered rnum from the file numbered fnum.

PPDEL fnum[,rnum]
Deletes record rnum from file fnum.

rnum=FFREE fnum
Returns the least record number not presently in use in file fnum.

PPDS fnum,rnum
Sets position of file fnum to rnum.

result=FSTATUS fnums
Returns the status of all files specified by the right argument. The result is a vector or matrix according to whether the argument is a scalar or vector. Columns are: (1) largest record number, (2) current position, (3) file size, (4) unused space, (5) lost space, (6) space not used because record sizes not divisible by 64, (7) 1 if coded file, (8) 1 if DA type, (9) 1 if absent record encountered by last read attempt.

PSTATUS
Prints status information (with descriptive headings) for all active files.

result=FAMES
Returns a matrix of user numbers and names for all tied files.

result=FNUMS
File numbers in use for tied files.

FRETURN fnums
Unties specified direct access files and erases specified indirect access files.

FUNTIE fnums
Unties files in right argument. This leaves a permanent copy.

ERASE *fnum*

Erases all files specified by right argument. Erasure affects active file and for DA type also affects permanent file.

'[*account] file-name [:passwd] [options]' *FTIE fnum*

Ties a file with specified options--*RO* for read only (other users can read at the same time), and *RM* for read-modify (another user can modify at the same time).

result+*CFREAD fnum*

Coded read. Result is character vector or numeric scalar--1 for end of record, 2 for end of file, 3 for end of information.

array *CFWRITE fnum*

The left argument is written to the coded file *fnum*. The argument should be a character scalar, vector, or matrix, or integers--1 to write end of record or 2 for end of file.

integers *CFPOS fnum*

Positions file. Operations indicated by first integer are: 0 for rewind, 1 for skip record, 2 for skip file, 3 for skip to end. Second integer for skip record or skip file may be included as repetition count.

jobname+*CSUBMIT fnum*

Submits the coded, indirect access file *fnum* as a batch job and erases the active copy.

FREAD 9 28

000
000

FREAD 9 3

RECORD 3

After the above steps, the user can store the file (using *FJNTIE 9*), an operation analogous to saving a workspace. The user could then sign off the system. The information in the file would remain intact and could be accessed or modified at a later time.

File limits. Individual file records are allowed to be as large as desired. However, user numbers have associated restrictions that may limit the total number of files, the total size of all files, the size of individual files, and whether the user can create direct access files. This information must be obtained from the installation or by executing the **LIMITS** control card (see Chapter 12).

Tied files. It is usually more convenient to use numbers within a program to identify a file rather than using the file name. All file operations require this file number. The number is tied to the file when the file is created using *PCREATE* or when a previously stored file is accessed using the *PTIE* function. Once a file has been assigned a number, the file is said to be tied. The file can be released by using the *PUNTIE* operation, the *PRETURN* operation, by erasing the file using *PERASE*, by signing off from APL, or by typing *CLEAR*. However, files remain tied when another workspace is loaded.

Accessing file functions. The functions described in this chapter are ordinarily stored under the user number *APL1* in the workspace *FILESYS*. Before file operations can be performed, the functions must be obtained from *APL1* by loading the entire *FILESYS* workspace or by copying selected functions from *FILESYS*. All functions in *FILESYS* are independent, and you need copy only those functions you intend to use. The following examples show various ways that copies of the file functions can be obtained.

```
)LOAD *APL1 FILESYS
)LOAD '*APL1 FILESYS'
)COPY *APL1 FILESYS .GRPPRIM (A group that excludes'
                             documentation)
)COPY *APL1 FILESYS PTIE PREAD
```

The file functions use the system function *QFI* to perform all file operations. The function *QFI* could actually be used directly, but it is usually more convenient to use the functions in the *FILESYS* workspace. Most of the functions in the *FILESYS* workspace are locked so that error processing will be more convenient. Users who wish to learn how to use *QFI* directly can discover all details about *QFI* by studying the definitions of the locked *FILESYS* functions below:

```
VA FCREATE B [1] A QFI 1,BV
VA FWRITE B [1] A QFI 2,BV
VZ+FPREAD B [1] Z+QFI 3,BV
VFPERASE B [1] B QFI 4V
VERDEL B [1] QFI 5,BV
VZ+FSTATUS B [1] Z+B QFI 6V
VZ+FNAMES [1] Z+QFI 7V
VZ+FNOMS [1] Z+QFI 8V
VFUNTIE B [1] B QFI 9V
VA PTIE B [1] A QFI 10,BV
```

```

VFPOS B [1] DFI 11,BV
VA CFWRITE B [1] A DFI 12,BV
VZ+CFREAD B [1] Z+DFI 13,BV
VA CFPOS B [1] DFI 14,B,BV
VZ+CSUBMIT B[1] Z-DFI 15,BV
VZ+PFHKE B [1] Z-DFI 16,BV

```

In addition to the basic functions in the *FILESYS* workspace, the workspace *FILES2* contains additional file functions that are based on the functions in *FILESYS* and perform more complicated operations.

Active and stored files. APLUM files are ordinarily *XRONOS* indirect access files unless the user specifies otherwise at the time of creation. This means that when the file is tied, the system makes a copy of the stored file. All reads and writes actually interact with this active copy. To save the file as a permanent stored file, an *FUNTIE* is required. Signing off from APL, typing *!CLEAR*, or a telephone disconnect (assuming the *RECOVER* command is not used) causes the active file to be erased. One advantage of having a separate active copy is that no damage can be done to a stored file if a series of file updates is not completed. For example, suppose that a program writes a record to indicate that a transfer of funds was made from one account to another on a certain date, then the program revises two records containing the balances of those accounts. If the program were to halt in the middle of the sequence of operations (due to a system problem or telephone disconnect), the transactions recorded in the file would be inconsistent with the balances in the file. This causes no problem when indirect access files are used because the inconsistent information is in a temporary file and the stored file is in the same state it was when it was tied.

For some applications that use indirect access files, it may be desirable to perform an *FUNTIE* and an *FFIE* at intervals of about every ten minutes in order to minimize the amount of information that would be lost in the event of a system problem.

Forms for file names and passwords. File passwords and file names must be composed of 1 to 7 of the letters A to Z and digits 0 to 9, must begin with a letter, and must not contain any embedded blanks. File names should be distinct from names used for other files or workspaces. Use of the same name will result in an error message when an attempt is made to untie the newly created file. (For a direct access file, the error occurs when *PCREATE* attempts to create the new file.)

Range for file numbers. File numbers can be any nonnegative integers not greater than 2^{17} .

Range for record numbers. File record numbers can be any nonnegative integers not greater than 2^{17} .

FILE SECURITY

A file is owned by the user who created it. The owner is allowed to alter the file in any desired manner, but the owner can control access by other users through the following controls:

1. The file category is ordinarily *private*. *Private* files cannot be accessed by other users unless their user numbers have been given explicit access permission by use of the KRONOS PERMIT command (see Chapter 12). Alternatively, the file can be assigned a category of *semiprivate* or *public*. Either of these categories allows other users to access the file if they know the password. The `QLIB` command will reveal to another user the names of files that are *semiprivate*, *public*, or that are *private* and have been explicitly made accessible to the other user. To make a file *public* or *semiprivate*, use the options `PU` or `S` when the file is created, or use the KRONOS CHANGE command to change the category. When the `QLIB` function is used with a file name, the result shows when the file was created, when it was last changed, and when it was last accessed. In addition, for *semiprivate* and *private* files the system retains the number of accesses and the time of the last access for each user of the file. This information can be displayed by use of the KRONOS CATLIST command (see Chapter 12).

2. The file can be given a password. Only users who know the password can use the file; however, the owner of the file is never required to provide the password. The password can be assigned when the file is created, or the password can be assigned or changed by use of the KRONOS CHANGE command (see Chapter 12).

3. The file mode can be used to control the type of operation another user can perform. For files created by APL (including workspaces) other users are ordinarily allowed to read the file (assuming the password and category do not exclude them) but are not allowed to alter or destroy the file. Other users can be given permission to alter the file by specifying the `WR` option (for write) when the file is created. For *private* files, this mode has no significance because when other users are given explicit access permission via the KRONOS PERMIT command, the permitted access

mode for each user becomes that expressed in the PERMIT command. For *semi-private* files, the general access mode is applicable to most users of the file, but an overriding access mode can be specified for individual users by use of the PERMIT command. For example, most users might be allowed to read the file, while a few selected users might be allowed to alter it. The general mode allowed for other users can be changed after the file has been created by use of the KRONOS CHANGE command. For APLUM files the mode should be write or read-modify, while for coded files it should be write or read.

4. Files can be accessed by other users through locked functions which can provide extremely general control over the permitted operations. For example, the locked function can prohibit alteration of the first five records of the file, or, it can prohibit adding records that are not vectors of 4 integers. The success of locked functions, as a security measure rests on preventing the user from learning the file name, the user number, or the password, and preventing him from accessing the file directly. To assure this, the locked function should not call other functions (except those local to itself) lest someone substitute a subversive function having the same name. In particular, `DEFI` should be used directly rather than using `FTIE`. (A subversive `FTIE` could print its arguments and thus reveal the file password). Also, `[` input should not be used while the file is tied, and the file should be untied prior to exit from the function. To ensure that the file will be untied, use `UNTRAP` to specify a trap line that will release the file prior to exit.

Note that the file category, password, and mode are independent restrictions on access by other users. Each of these further restricts the type of access permitted to others. Unless different options are specified when the file is created or the controls are changed, the APLUM system selects *private* as the file category, assigns no password, and selects read or read-modify mode (depending on whether the file is coded or APLUM type, respectively) so that other users may only read the file.

APLUM FILE OPERATIONS

Sequential file operations. The file operations that ordinarily require a record number can also be used without specifying the record number. When this is done the record number used is the current file position (available in the result of `FSTATUS`). The file position can be reset using `FPOS` and is incremented by each

successful read, write, or deletion. When a file is tied or created, the position is initially zero. For example:

```
'XPAY'PTIE 5 (The file position is zero.)
Z*PREAD 5 (Record 0 is read; the position becomes 1.)
X FWRITE 5 (Record 1 is written)
Y FWRITE 5 (Record 2 is written)
W FWRITE 5 (Record 3 is written)
```

When a record number is provided for the operation, the file position will be set one greater than that number if the operation succeeds.

File create: 'file-name [:passwd] [/options]' FCREATE fnum
The file create function can be used to create a file and specify options about the type of file. When the file is created, it is tied to the file number fnum. In addition to the name of the file, the left argument may include the password the file is to have. Examples of file creation follow:

```
'FILE1' FCREATE 11 (A file named FILE1 with 11
                   as its number.)
'FILE2: SESAME' FCREATE 2 (A file with SESAME as its
                           password.)
```

The list of options can include any of the following separated by spaces: DA, C, WR, S, or PU (to specify direct access, coded, write mode, semiprivate, or public). The option RM is appropriate for APLUM files, while RD is the corresponding option for coded files. These are discussed in other sections of this chapter.

File write: array FWRITE fnum[,rnum]
The FWRITE function writes its left argument on the file having fnum as its number as the record having rnum as its record number. This will replace any existing record in that file previously having that record number.

File read: result+PREAD fnum[,rnum]
The PREAD function reads from the file having fnum as its file number that record having rnum as its record number. If that record does not exist, an empty numeric vector is returned, and the file status (see FSTATUS) will indicate that the last read attempt encountered a nonexistent record.

File record delete: FRODEL fnum[,rnum]
The FRODEL function deletes the record rnum from file fnum. If the record was absent already, nothing is done (except that the file position changes) and no error results.

Free record number: *rnum+FFREE fnum*

The *FFREE* function returns the first free (unused) record number for file *fnum*. This is a useful way to select the record number for a new record when the application does not require a particular ordering of the records.

File positioning: *FPDS fnum,rnum*

The function *FPDS* sets the position of the file identified by *fnum* to record number *rnum*.

File status: *result+PSTATUS fnums*

The file status function returns various information about the condition of files identified by file numbers in the right argument. If the argument is a vector, the result is a matrix having a row for each file number in the right argument. If the argument is a scalar, the result is a vector of information about the single file. The columns of the result contain:

<u>Column</u>	<u>Contents</u>
1	Largest record number currently in use or '1' if the file is empty.
2	Current file position.
3	File size in words.
4	Unused space in words.
5	Lost space in words.
6	Space not used because of record sizes not being divisible by 64. (This space is called "tails" because it resides at the tail ends of physical record units.)
7	0 if APLUM type file, 1 if coded type file.
8	0 if indirect access file type, 1 if direct access file type.
9	0 if last read attempt succeeded, 1 if the record was absent.

Note that only columns 7 and 8 are meaningful for coded files. All columns will be zero if the file is not tied.

The largest record number does not take account of records that have been deleted. That is, the largest record number is the largest number currently in use for records that actually exist. The largest record number is convenient to know when adding a new record to the file. Adding 1 to the largest record number gives a safe record number to use to append a new record.

Print status: PSTATUS

The *PSTATUS* function prints the information returned by *FSTATUS* *FNUMS* along with the file names. The information is given in a descriptive format and is thus a convenient way to discover the status of all tied files if you do not remember the meanings of the columns in the result from *FSTATUS*. The following example illustrates the format used.

NAME	NUMBER	LAST R	POS	SIZE	UNUSED	LOST	TAILS
CONTIME	14	8	0	758	64	0	397
LIB	2	1	14	256	0	0	98 DA
*A123456 SYSGEN	45	COLLED FILE					
REPMAN7	1	94	0	80384	2496	7744	3233 DA

File names: result=FNAMES

The *FNAMES* function returns a matrix of names (and user numbers) of files currently tied. The number of columns in the matrix is always 16. For example,

```
FNAMES
SAMPLE1
ALGEBRA
*A123456 FILE1
```

File numbers: result=FNUMS

The *FNUMS* function returns a vector of numbers in use for tied files. The order is the same as the order of file names in the result from *FNAMES*.

File untie: FUNTIE fnums

The *FUNTIE* function unties all files for which their file numbers appear in the vector or scalar right argument. This produces a permanent stored copy of each file. The new permanent copy will replace any previously existing file having the same name, unless the active file was newly created. To untie a newly created file when the same name is already in use for another stored file, first use *QDROP* to remove the old file. If any of the files specified in the argument is not tied, nothing is done and an error message results. To untie all tied files, use *FUNTIE FNUMS*.

File return: FRETURN fnums

The *FRETURN* function behaves as *FUNTIE* for direct access files and behaves as *PERASE* for indirect access files. This frees the number of a currently tied file for other uses with a minimal impact on stored files. The use of this function is recommended for cleaning up any files that may have been accidentally left tied. File numbers in the argument that are not in use for tied files are ignored.

File eraser: *PERASE fnum*

The *PERASE* function erases the active copy of the file but leaves any stored copy of the file. (See the section on direct access files for exceptions.) To remove a stored copy, use *[EPCP]*.

File tie: '*file name [:passwd] [,options]*' *FTIE fnum*

The *FTIE* function gives the number *fnum* to the previously stored file having the indicated name. If no previously stored file having that name is found, an error message is given and no file tie results. If a user number is given, the stored file is sought under that user number rather than the one used when signing on to the system. The password need be given only if a another user number was provided and a password was given to the file when it was created using *PCREATE*. Examples using *FTIE* follow:

```
'FILE5' FTIE 7           (A user ties one of his own files.)
'*A0Q1234 FILE6' FTIE 8   (A user ties a file belonging
                           to another user.)
'*A123456 FILE7 :SESAME' FTIE 9
```

Note that the options *DA* and *C* (for direct access or coded files) must not be provided to the *FTIE* function. These options are chosen when the file is created and can be altered only by making a copy of the file. If the file number or file name is in use for another tied file, an error message results. The list of options can include either of the options *RD* or *RW*. These options are discussed in later sections.

SPECIAL CONSIDERATIONS FOR CODED FILES

Coded files are the standard type of file on the KRONOS system for information interchange between programs, card readers, printers, and so forth. APLUM can access any coded files provided they contain lines no longer than 1280 characters. Coded files are essentially intended for sequential access; replacement of records, except at the end, is not practical. Instead, such changes would ordinarily be made by copying the file and making the changes as the new file is produced.

Coded files consist of lines (essentially vectors of characters) which can be separated into groups by end of record marks. These groups can in turn be separated by end of file marks. At the end of the file is an end of information mark. The characters in a line of a coded file are restricted to 63 characters. The 256 APLUM characters are translated into these 63 characters as shown in Appendix C. Briefly, the letters *A* to *Z* become *A* to *Z*, all symbols with approximate equivalents for an ASCII printer are translated into those equivalents, and all others become *@*. When translating from the KRONOS character set

to APL characters, all symbols are represented by equivalents, and # is represented as \$ (the symbol used for illegal characters).

The functions *FTIE*, *FUNTIE*, and *FRETURN* have essentially the same meanings for coded files as for APLUM files. However, special functions must be used for reading, writing, and repositioning coded files.

Creating a coded file. A coded file can be created using *CFCREATE* by including *C* as an additional parameter. For example,

```
'PRINT :XXX/C'CFCREATE 5
```

Coded read: *result*+*CFREAD* *inum*

The result returned by *CFREAD* is a character vector containing the next line from the file, or if an end of record, end of file, or end of information was encountered, the result is the scalar integer 1, 2, or 3, respectively. The file position changes after each read so that the next read will give the next line of the file. The *FREAD* function cannot be used in place of *CFREAD* with a coded file.

Coded write: *array* *CFWRITE* *inum*

The left argument to *CFWRITE* is written at the current position of the file. The left argument must be a character vector, scalar (which is treated as a one-element vector), or matrix, or a scalar or vector containing the integers 1, 2, or 3. A character scalar or vector produces one line in the file, whereas a matrix produces one line for each row of the matrix. Trailing blanks in a line are removed. The integers 1 or 2 produce an end of record mark or end of file mark, respectively. A vector of integers can be used to produce a series of these marks. The file position is altered after each write so that subsequent writes will add information after that produced by the present one. Anything written to the file is automatically followed by an end of information mark. This has the effect of truncating the file if the write was not performed at the end of the file. The function *FWRITE* cannot be used for a coded file in place of *CFWRITE*.

Coded file positioning: *integers* *CFPOS* *inum*

The function *CFPOS* repositions the file according to integers in the scalar or vector left argument. The first element in the left argument indicates the action to be taken, and the optional second element may contain a repetition count.

Operation	Value
Rewind	0
Skip record	1
Skip file	2
Skip to end	3

The rewind operation positions the file at its beginning. The rewind and skip-to-end do not allow use of a repetition count. For the skip record or skip file operations, the repetition count may be negative to skip towards the beginning of the file. If no repetition count is given, a count of 1 is assumed. The skip record operation counts end of file marks as records. The skipping never goes past the end of information mark or the beginning of the file, even if the repetition count has not been satisfied.

Batch job submission: Z-CSUBMIT fnum,type

The coded file *fnum* is submitted as a batch job. The *type* may be 0 if batch output produced by the job should be discarded, or 1 if it should be printed or punched at the central batch site. The file must be a properly constructed job file (see KRONOS 2.1 Reference Manual). In particular, the first two lines must be a job card and account card. The file must not be direct access type. If the operation is successful, the active file vanishes as if *PERASE* had been used. The result returned is the job name assigned to the job. This name can be used with *KRONOS ENQUIRE* command (see Chapter 12) to determine whether the job has completed. Note that the number of concurrently executing deferred batch jobs allowed for a given user number is controlled by the system. The number can be determined by use of the *LIMITS* control card (see Chapter 12).

SPECIAL CONSIDERATIONS FOR DIRECT ACCESS FILES

A direct access file differs from an indirect access file in that all operations interact with the permanent file itself, not with an active copy. This has both advantages and disadvantages. One advantage is that a copy of the entire file need not be made by the system when the file is tied. One disadvantage is that a program can stop executing due to a system problem in the middle of a series of file writes, and the stored file can end up with contradictory information. Another disadvantage of direct access files is that write operations take a little longer (because the *APLUM* system does less buffering of information due to the risk of a system problem freezing the file in a temporary state).

To create a direct access file, include the parameter *DA* in the left argument to *PCREATE*. A direct access file may also be a coded file if desired--these two options can be chosen independently. The following are examples of direct access file creation:

```
'FILEX/DA' PCREATE 4  
'FILEY: X12/DA S HR' PCREATE 5  
'FILEZ/DA' PCREATE 6
```

All operations with direct access files take the same form as for indirect access files, but because of the differences

between the two file types the file tie, untie, and file erase operations behave differently: A file tie to a direct access file does not make a copy of the file. An untie does not create the permanent copy, it merely releases the file number for use with other files and releases the file itself for access by other users. An erase removes both the active and stored copy of the file because they are the same thing. In addition,)CLEAP or a telephone disconnect cause an automatic FUNTIE of a direct access file (thus leaving a stored file) whereas an indirect access file would be erased.

If an telephone disconnect occurs, the file remains tied for 10 minutes. The operations that were in progress can be continued by use of the KRONOS RECOVER command (Chapter 12). However, an attempt to sign on without using the RECOVER command will leave the file tied until the 10 minute period is over, possibly causing an error message indicating the file is busy.

SYNCHRONIZED FILE OPERATIONS

At present, it is not very practical for two users to update a single file at the same time. With an indirect access file the two users are actually updating separate copies of the same file, and whichever user unties the file last will create a stored file with his updates, but will replace any stored file just produced by the other user. The KRONOS operating system does not allow two users to be tied to the same direct access file in write mode at the same time, so no conflicts can occur, but an error occurs if a second user attempts to tie the file. However, users can tie a direct access file in read mode (which allows other users to read the file at the same time) or read-modify mode (which means the user desires only to read the file but has no objection to another user writing to the file at the same time). To tie a file in read mode or in read-modify mode, include RD or RM (but not both) in the left argument to the FTIE function. For example,

```
'FILE1/RD'FTIE 9 (Read mode.)
```

```
'FILE2; SECURE/RM' FTIE 10 (Read-modify mode.)
```

These modes have meaning only for direct access files and must not be used unless the file is direct access. Read mode can be used for APLUM or coded files while read-modify mode is allowed only for APLUM files.

FILE EFFICIENCY

Although many users need not concern themselves with the information presented here on file efficiency, users of very large files will find this information important. Use of a few

fairly simple techniques can result in improved speed and reduced storage requirements.

First of all, each APLUM file has an initial size of 64 words used for a table of available space. In addition, one word is required for each record number up to the last record number in use. This space is allocated in multiples of 64 words. These two factors combine to make it inefficient to store many files with only a few records in each rather than one file with many records. Also, it is inefficient to leave large gaps between record numbers as the unused numbers require an average of one word each.

Indirect access files grow in multiples of 64 words, but direct access files grow in multiples of a logical track (usually several thousand words, depending on the storage device used). There is consequently a considerable space advantage to using indirect access files for files smaller than several thousand words.

File records require 2 words more than they require in the workspace, and the space for records is allocated in multiples of 64 words. However, large records of character or logical type require only about half as much space in the file. Because records require multiples of 64 words, there is some saving in space if many little arrays can be packed together and written as a single record.

When records are erased or replaced by records of a different size, the APL system keeps track of any unused gaps in the file where records can be placed in the future. The total amount of this space in words is in column 4 of the result returned by the *STATUS* function. It may happen that the number of gaps exceeds the size of the table, in which case the smallest gap is removed from the table. This results in a certain amount of space becoming unusable, and the total amount of this lost space is in column 5 of the result returned by the status function. Lost space can also result in a direct access file if a telephone disconnect or system problem prevents the file from being untied (*[T]M'SYSTEM'*, *[T]M'ABOFT'*, and *[T]M'OFF'* untie files properly), and if the *RECOVER* command is not or can not be used. All lost and unused space can be recovered by copying all records to a new file, last record first, and using the new file as a replacement for the old one. Because each record occupies a multiple of 64 words, some space is generally left unused. This space is returned in column 6 of the result from *STATUS*.

Details of the space required for coded files can be found in the KRONOS 2.1 Reference Manual. Coded files have a speed advantage over APLUM files when the information is accessed sequentially, the records are small, and the limitations of the 63 character set are not restrictive.

INTEGRITY OF DIRECT ACCESS APLUM FILES

File integrity refers to the ability of a file to retain internal consistency. Some file access methods render a file practically useless if a program operating on the file does not complete properly (due to a flaw in the program or a system problem). Every effort has been made in the design of the APLUM file system to minimize the chance of such damage.

All alterations to an APLUM file are performed immediately and thus occur in exactly the order requested. When multiple files are being updated, one file will not be several transactions ahead of another. A checksum is computed for each file record so that if the storage device corrupts the information and is unable to detect the error, the error will still be detected by the APLUM system. A system halt, program halt, or telephone disconnect will leave the file in a satisfactory state except that in the rare event of a system halt requiring a level zero deadstart within a minute of extending a direct access file, there is some chance of damage to newly created or replaced records.

File damage will cause an error message to be printed at the time it is detected. The damage will usually affect only one record of the file. If the file cannot be reconstructed, installation personnel can assist with restoring the file to its state the last time files were dumped to magnetic tape.

Note that a telephone disconnect or system problem that results in failure to untie the file may cause the information on file space utilization (unused space, lost space, and tails) to be incorrect. This does not hinder utilization of the file and can be corrected by copying the file.

FILE EXAMPLES

The following sample functions taken from the workspace *FILES?* under user number *APL1* illustrate simple file operations. The first function, *FCOPY*, can be used to copy an APLUM file. Such a copy might be made to convert the file from indirect access to direct access form or to compact the file by minimizing unused space. The left argument should be the character argument required to tie the old file, and the right argument should be the character argument required to create the new file. Note that the first line illustrates a simple way to select a file number that is not already in use.

```

FCOPY[0]
9A FCOPY B;P;K;I;J
[1] A FTIE I←1+I/C,FNUMS
[2] B FCRATE J←I+1
[3] K←(FSTATUS I)[1] a GET LARGEST RECORD NUMBER
[4] L1:→(K-0)/L3
[5] P←PREAD I,K a READ RECORD K FROM FILE I
[6] →(FSTATUS I)[9]/L2 a IF ABSENT RECORD
[7] P FWRITE J,K a WRITE RECORD K TO FILE J
[8] L2:K←K-1
[9] →L1
[10] L3:FUNTI I,J a UNTIE BOTH FILES
[11] 'COPY COMPLETE'
9

```

The next function is useful for listing a coded file. The right argument may be the name of a stored file or the number of an active file. If a name is given, the file is tied, listed, then untied. If a number is provided, the file is listed beginning at its current position and is left tied.

```

VCLIST[0]
VCLIST B;K;L
[1] →(0-0\0pK+B)/L1 a IF FILE ALREADY TIED
[2] B FFILE K←1+I/O,FNUMS
[3] L1:L←CPREAD K
[4] →(0-0pL)/L2 a SCALAR INDICATES SPECIAL MARK
[5] L
[6] →L'
[7] L2:→L3+2×L-1
[8] L3:' END OF RECORD-'
[9] →L'
[10] '-END OF FILE '
[11] →L1
[12] '-END OF INFORMATION-'
[13] FUNTI(0-0\0pB)/K

```

The next two functions are useful when a file is too large to list at a terminal but it is necessary to learn the general structure of the file. The function FMAP prints the structure of an APLUM file, and the function CMAP prints the structure of a coded file. Both functions allow a character argument or a numeric argument in the same manner as CLIST. If the file is already tied (for numeric arguments) the mapping begins at the current file position. FMAP prints record numbers and the types (C or N for character or numeric) and shapes of records that exist, or ABSENT for absent records. CMAP prints the number of lines in records and prints FOR, EOF, or EOI when an end of record, end of file, or end of information is encountered.

```

VFMAP[ ]?
VFMAP B,K;P
[1] +(0=0\0pK+B)/L1 * IF B IS NUMERIC
[2] B FTIE K+1[/0,FNUMS
[3] L1:+(1*1+FSTATUS K)/L2 * IF FILE NOT EMPTY
[4] 'NO RECORDS'
[5] +0
[6] L2:'NUMBER, TYPE, DIMENSIONS'
[7] L3:+(1/2+FSTATUS K)/L5 * IF FINISHED
[8] P-FREAD K
[9] +(FSTATUS K)[9]/L4 * IF READ FAILED
[10] ~1+(FSTATUS K)[2];0 1 0\CH'[1+0=0\0pP]:pP
[11] +L3
[12] L4:~1+(FSTATUS K)[2];' ABSENT'
[13] +L3
[14] L5:FUNTIME(0=0\0pB)/K
?

```

```

VCMAP[ ]?
VCMAP B;K;P;C
[1] +(0=0\0pK+B)/L1 * IF B IS NUMERIC
[2] B FTIE K-1[/0,FNUMS
[3] L1:C-0
[4] L2:+(0=ppP+CFREAD L)/L3
[5] C+C+1
[6] +L2
[7] L3:+(C=0)/L4
[8] C:' LINE',(C=1)/'S'
[9] L4:'EO','REF'[P]
[10] +(P<3)/L1
[11] FUNTIME(0=0\0pB)/K
?

```

Chapter 10. APL Public Libraries

The standard APLM release includes the following workspaces stored under the user number APL1:

APLNEWS News about the changes in the APLM system as well as a list of reported bugs and requests for system changes.

WSFNS Workspace utility functions. Includes functions *SHORT*, *LONG*, and *SHORTSAVE* for saving a workspace without *OFF*, *ON*, *Q*, *R*, *I*, and *T*.

FILESYS File system functions.

FILES2 Contains functions from *FILESYS* for primitive file operations as well as additional functions for more elaborate file operations.

PLOTFRS Function to produce an X Y plot of multiple sets of data on a standard terminal.

CATALOG A guide to workspaces in the APL public libraries.

STP1, *STP2*, *STP3*, *STP4*, *STP5*, *STP6*

A collection of mathematical and statistical programs developed by K. W. Smilie of the University of Alberta. Primary documentation is in *STP1*. Capabilities include: descriptive statistics, regression and correlation analysis, analysis of variance, linear programming, and critical path.

To learn how to use any of these workspaces, type a command of the form *LOAD*'*APL1 CATALOG' and then type *DESCRIBE*.

APL PROGRAM LIBRARY STANDARDS

It is suggested that installations reserve the user numbers *APL1* to *APL999* for APL public libraries. Although these user numbers need not be defined in the system, they should not be used for other purposes. It is suggested that programs placed in these public libraries be of fairly general interest so that users will find it rewarding to browse through the various workspaces. Workspaces of interest only to a specialized group or course should be stored elsewhere.

Programs placed in the public libraries should be well documented. The available documentation may be entirely in the workspace or partly in the workspace and partly in a manual. In any case, the documentation should be readily available. The advantage of having the documentation in the workspace is that it will be immediately accessible. The disadvantages are that the documentation is slow to print and therefore tedious to read, and the format of the documentation is constrained by the APL character set. Generally, the amount of documentation determines whether it is practical to put the documentation in the workspace.

Documentation in the workspace should consist of functions or variables that describe the workspace. The documentation should be able to be printed with a standard APL terminal and should print within a standard 65 column page width. The following documentation variables or functions are suggested. Typing the name of the function or variable should cause the information to be printed.

ABSTRACT. Should contain a brief description of the contents of the workspace.

DESCRIBE. This should give the user further details than that provided in the *ABSTRACT*. This should print the names of all functions intended for the user to use as modules along with a short description and names of related *HOW* functions (see below). If groups are defined in the workspace, describe them and their purposes.

HOW functions. If a function has the name *NAME*, detailed documentation of that function should have the name *NAMEHOW*. There is no point in giving a line-by-line description of the function. The APL program is already an excellent description of the separate steps. The *HOW* function should tell what the function does and how to use it as a module. In some cases it should outline major steps in the processing and describe the method used. References might be appropriate. Special limitations of the function should be discussed.

SOURCE. Should give the author's name, an inquiry name, and an inquiry address. The date when the workspace was contributed should be included.

CHANGES. Changes should be documented by a function or variable having a name of the form *CHANGES092678* (so that the name includes the date of the changes).

GRPDOC. The group (locked matrix of names) *GRPDOC* should include names of all documentation variables and functions so that the user can readily erase them to make more space available in the workspace or reduce disk storage charges.

Even when most of the documentation is in a separate manual, the following variables or functions are required: *ABSTRACT*, *SOURCE*, *GRPDOC*, and *DESCRIBE*.

Chapter 11. Optimization of APL Programs

This chapter discusses some of the techniques that can be used to make APL programs perform better and run with lower demands on computer resources. It may seem out of place to discuss efficiency in an APL manual--after all, APL should free the user from being concerned with the nature of the particular computer being used--but the techniques discussed here may yield efficiency improvements as large as a factor of a hundred. To neglect discussing efficiency could leave many users with the mistaken impression that APL cannot perform well enough to be used for their problems.

Often, the question of efficiency calls to mind the fanatical programmer who constructs a program he considers efficient but who in doing so produces a totally incomprehensible collection of operations. It should be remembered that for many programs the programming time is so great that the only kind of efficiency worth considering is the sort that makes the program easy to understand, free of errors, and easy to change. Fortunately, a simple program is usually an efficient program. However, when improving the performance of the program does not coincide with simplifying it, the optimization should not be applied unless it is very important for the program to perform well.

As a very blatant example of misguided optimization, consider the following statement:

```
K←1,0pP←0,⌊/L←1Q←pP
```

This statement was probably contrived by someone who believed that the most efficient program was the one that required the smallest number of lines. The fact is, execution proceeds from one line to the next very rapidly compared to the time required to perform the extra steps needed to fit the operations in one line. The following statements are a more straightforward way to achieve the same results:

$$K \leftarrow 1$$

$$L \leftarrow Q + pR$$

$$P \leftarrow 0, Q$$

One way to estimate the relative time required for an expression is to count the number of operations required. (This method is fairly valid when the number of elements in arrays is less than about 20.) For this method of estimation, specification is not counted at all (it takes relatively little time). The one line version totals 6 operations while the three line version requires only 3 operations. The efficiency expert who wrote the one line version devoted extra time to adding three operations, which double the time required for execution. The one line version is harder to understand, is more likely to contain errors, and when changes are made, the rest of the line hinders revision. The one liner is thus a poor example of efficiency in all respects.

At this point it must be stated that much of the information in this chapter is relevant only to this particular APL system. Also, it may occur that something that is particularly slow now will become particularly fast in later versions of the system. Other versions of APL on other computers will often show quite different characteristics. In fact, according to Paul Berry, the popular belief that one line programs are more efficient is based on a system for which this is true. A version of APL on the IBM 1130 actually requires considerable time to change from one line to the next because only one line at a time is kept in main memory. Although very few present users of APL ever used that particular system, its influence persists.

STORAGE REQUIREMENTS

Although the APLUM system allows a workspace of up to about 90,000 words (provided the user is validated to use that much main memory and the installation has that much), equivalent to 675,000 8-bit bytes, there are practical reasons to keep a workspace smaller. The KRONOS operating system uses computer resources much more effectively when it runs programs requiring minimal amounts of central memory. Also, the "response time" for an interactive program to respond to a command requiring a trivial amount of processing increases somewhat with central memory requirements. In addition, minimizing storage requirements improves the chances that the same program will be able to run under another version of APL or on a computer with less central memory.

The vector `OWA` contains information about the memory currently in use for the APLUM interpreter and the active workspace. The field length is the amount of memory space currently in use. The APLUM system manages that memory space and at any given time some of the space may not be in use for

functions, variables, and other information kept by the APLM system. The APLM system evaluates storage requirements from time to time and resets its actual field length according to current needs. The user can set `MA` to specify the maximum and minimum field lengths to be used. Increasing the maximum and minimum field length generally reduces the central processor time used by APLM to reorganize its storage, but as discussed previously, reduces the operating system efficiency. As a general rule of thumb, leave the minimum field length at its normal value, and set the maximum field length large enough to avoid `WS FULL` plus a little extra to prevent frequent storage reorganization. Incidentally, referencing the value of `MA` in a statement causes the APLM system to reorganize its storage, so programs should not alter or read the value of `MA` too often or performance will be degraded.

Obvious techniques for minimizing storage requirements include using algorithms that minimize temporary storage, using local variables and local functions to assure automatic erasure of unneeded objects, and using `EX` to erase other functions that are no longer needed. `EX` can also be used to erase variables, but respecification (e.g., `A←''`) is faster. Files can be used to store functions and variables until they are required. `LOAD` can be used to load another workspace of functions and variables. Any variables that must be communicated from one workspace to the next can be placed in files--files remain tied when another workspace is loaded. Of course, any of these techniques can be overdone. Do not let the time spent performing these operations outweigh the time they save.

The space in words required for an APL array `A` is

$$2 + (p \cdot A) + ((n/p) \cdot A) + D$$

where `D` is the number of elements packed per word--1 for floating point values, 4 for characters, and 32 for logical. Clearly, there is an advantage to using the internal logical representation if the values are ones and zeros. The system does not always use the logical representation when it could. For example, the scalar constants 1 and 0 are floating point, and `1+0` is floating point. However, the following functions always produce a logical result: `A∧B`, `A∨B`, `A∧B`, `A∨B`, `A=B`, `A≠B`, `A<B`, `A≤B`, `A≥B`, and `A<B`. Also, the functions that restructure or rearrange their arguments always preserve the same type of representation, so `Wp0` is floating point, while `Wp1 0` is logical (because vector constants consisting of ones and zeros are packed as logicals). To assure that a result is logical, apply `1=` to it.

Expressions like `A+B+C+100` do not cause three copies of `100` to be produced. Actually, only one copy is kept. However, subsequently altering an element of `A`, `B`, or `C`, (e.g., `A[1]←9`) will cause a separate copy to be made. Similarly, arguments to functions are not actually copied unless an attempt is made to

alter them using indexed specification. Unlike most other APL systems, using function arguments rather than global variables incurs no storage penalty.

Storage requirements for programs are too complicated to discuss in detail. As a rule of thumb, unless you make a special effort to put a lot on each line, figure that an average statement takes about 10 words of storage. The first time a statement is executed it is converted to an internal form for more efficient execution. In the internal form the function almost always requires somewhat more space. The storage overhead per line of a function averages about 3.5 words for lines without labels and 4.5 words for lines with labels.

In version 2.12 of APLUM, function definition mode and system commands are performed by functions written in APL. When a user types `)expression` or `^expression` the system translates these into `[SY]'expression'` and `[PD]^expression'`, respectively. The functions `[SY]` and `[PD]` are stored in the so-called "clear" workspace. In addition, some of the mixed functions are written in APL and take space in the "clear" workspace. It may be desirable to erase these if they are not required. For example, anyone who has mastered the equivalent system functions can eliminate `[SY]` from his workspaces. Similarly, after a workspace has been developed, `[PD]` can often be eliminated without hindering use of the programs in the workspace. To minimize disk storage use of the programs in the workspace. To minimize disk storage charges `[PD]` and `[SY]` can be erased before saving the workspace. In the workspace `WSPNS` in library `APL1` there is a function named `SHORT` that can be used to erase all of these functions, and there is a function named `LONG` that will copy in all of the functions. Future system improvements will eliminate all of these functions from the clear workspace.

The APL system keeps a "symbol table" in the workspace containing all names of functions, variables, and labels. Once a name has been used (even if the use resulted in a `VALUE ERROR`) the name remains in the symbol table. The space used by names that are no longer needed can be recovered by copying all objects into a truly clear workspace. The recommended procedure is:

```
[LOAD '*APLO EMPTY' (A clear workspace.)
[ENV←0 (Copy global objects.)
(1 2 3 4 [NAMES 'OLDWS'])COPY 'OLDWS'
[SAVE 'NEWWS'
```

This procedure will also recover space in workspace areas other than the symbol table in some circumstances.

Space can be conserved in the symbol table by using names consisting of a single symbol whenever possible. Space can also be conserved by using the same name in several functions for local variables or labels. A common convention is to use the letters `A` to `Z` for local variable names and use `L1`, `L2`, and so forth for labels.

CENTRAL PROCESSOR TIME

For many programs the main optimization problem is to minimize central processor time. First of all, one of the primary determinants of central processor time is the appropriateness of the algorithm used. The algorithm should be appropriate to the data to be processed and appropriate to APL. Computer literature is filled with algorithms that are "efficient" for other languages but which perform miserably in APL. Often a straightforward translation of a program from another language gives a program that performs poorly because it fails to take advantage of the more powerful APL functions.

For most operations in APL the time required for the operation can be separated into a per-element time required to process each element of the arguments and result plus a setup time required for interpreter overhead, to check the arguments for compatibility of dimensions, to compute the result dimensions, and allocate space for the result. The time per element varies considerably with the complexity of the operation. The sine function, for example, requires far more time per element than addition. The time also depends some on the way the values are stored; operations defined only for logic values perform better if their arguments are internally represented as logical type, and arithmetic operations are faster for the floating point internal type. The setup time varies far less from function to function than the time per element.

For many functions the setup time is on the order of 20 times as great as the time per element. This means that the setup time is negligible when thousands of elements are to be processed, but the setup time constitutes about 95 percent of the time when only one element is being processed. For most programs, the setup time limits speed more than the time per element. Thus the first step to optimization is to minimize the number of operations to be performed. For example, if ρX is used many times in a function, it would be worthwhile to assign the value of ρX to a variable (assignment requires negligible time). Often a branch statement can be added to skip steps that are not required except in special cases.

When the arrays used have a large number of elements, the operations should be chosen to minimize the number of elements processed. For example, if V is a vector of 5000 characters, a few elements can be selected from V using $V[N+1]$ (which might process about 5000 elements) or using $V[J+K]$ (which would process only a few elements). The second approach is much more efficient. Similarly, rather than extending a vector by concatenating one element at a time, it might be preferable to extend it with a large number of elements and then respecify the elements one at a time using indexed specification.

It is commonly believed that APL branching for looping is slow. Actually, looping is fairly fast by itself but is usually a sign that the program is performing operations one element at a time - the amount of time required is mainly due to the number of operations being performed. Actually, looping is sometimes a very efficient way to perform an operation, especially if the number of iterations required for normal cases is small and the alternative requires more operations than are used in the loop.

The evaluation of constants other than 1, 2, -1, 0, .5, and ' ' takes about half as much time as the setup time for functions. Some time can be saved by using variables to store the values of commonly used constants other than the common constants just mentioned.

On some APL systems central processor time can be saved by concatenating output together and then printing it in a batch rather than as it is generated. However, on the APL/M system it is more efficient to print the output as it is produced.

The following chart gives approximate timings for various operations. Be forewarned that these timings are approximate and will vary with the version of APL/M in use, the particular computer used, and the internal workspace configuration. Times are expressed in terms of T , the time per element for addition.

<u>Time range</u>	<u>Operations</u>
0 to T	Time per element for $A+B$, $A\vee B$, and $\sim B$ for logical internal representation
T to $5 \times T$	Setup time per statement to be evaluated Time per element for most scalar and mixed functions
$5 \times T$ to $25 \times T$	Time per element for complicated functions such as $A \circ B$, $A \oplus B$, $A \uparrow B$, and $A[B]$ Time required for an unnecessary set of parentheses in a statement Time required to evaluate a constant other than 0, 1, 2, .5, -1, and ' ' Extra time per local variable for a function call $\sim B$ $A+B$
$25 \times T$ to $125 \times T$	Call to a user defined function with a few local variables Setup time for primitive functions

Chapter 12. KRONOS Features for APLUM Users

This chapter discusses a few KRONOS commands of interest to users of APLUM. The discussions cover only the more important details. Further information can be found in the KRONOS 2.1 Reference Manual or in the KRONOS Time-sharing User's Manual. Most of the commands discussed can be used as timesharing commands or batch job control cards. However, they cannot be used while in APL. Use the commands before issuance of the APLUM command, or use `[TN'SYSTEM'` to leave APL to use these commands. Note that none of these commands allow embedded spaces.

HELLO

The HELLO command allows you to sign on again with a different account number.

BYE

The command BYE is the correct way to sign off the system when not in APL. This is equivalent to the APL command `[PW'OFF'` or `)OFF`.

RECOVER,number

The RECOVER command can be used to return to the state just before a disconnect or system malfunction occurred. The use of this command prevents loss of the active workspace or active files. The command is allowed only when the system prints RECOVER/SYSTEM at the end of the sign-on procedure. If you have already proceeded beyond that point and wish to initiate recovery, type HELLO to begin the sign-on procedure anew. The number you provide in the RECOVER command should be the terminal number that was printed after the previous sign on. (That is, the terminal number in effect for the session that terminated

abnormally.) After you type the RECOVER command, the system may print RECOVERY IMPOSSIBLE, which indicates that the system malfunction was too serious to allow recovery, that too much time has elapsed (recovery information is retained for ten minutes), you signed on with a different user number, or that you gave an incorrect terminal number. When the RECOVER command is successful, the recovery information is destroyed and the system prints various information about the status at the time of disruption. Press the RETURN key to continue (or type STOP to exit from APL). The recovery is sometimes imperfect. Some output may be lost, and the next input request may cause a question mark to be printed, and any special APL symbols used in the input may be translated incorrectly. Do not perform the recovery on a different type of terminal from that in use when the disruption occurred or the APL system will translate input and output incorrectly for that terminal.

LIMITS

The LIMITS command causes validation limits for the account number currently in use to be printed. Any numbers in the output that are followed by a B are expressed in octal (base 8). The APL functions base-value and represent can be used to convert between octal and decimal. For example, 70000B can be converted to decimal using `B17 0 0 0 0`, and 32768 can be converted to octal using `(6p8)r32768`. The following are the limits that are important to APLUM users:

- TL = CPU time limit in 10's (octal) per session. Append a zero to the right of the number to find the CPU time limit in octal seconds. In addition, there may be a smaller time limit per session. This other time limit per session can be overridden by using the SETTL command or by using the T,number command after a *TIME LIMIT* error occurs. If you have consumed your entire CPU time limit for the session, you can use the KRONOS HELLO command to get a new CPU time allotment, or hang up, sign on again, and use the KRONOS RECOVER command.
- CM = Maximum central memory field length. Append two zeros to the right of the number to find the central memory limit in octal words. Note that a more stringent restriction can be imposed on all timesharing users by the computer operator. This second restriction may vary according to the time of day.
- = The number of jobs allowed for the given user. The CSUBM.? function (see Chapter 9) is not allowed to submit additional jobs if the total number of jobs for that account number already equals or exceeds this parameter. The count of jobs includes the program attempting to use the CSUBMIT function.

FC = Maximum number of stored indirect access files allowed.

CS = Total storage in PRU's allowed for all stored indirect access files. (One PRU is 64 words or 640 six-bit bytes.)

FS = Maximum size in PRU's allowed for individual stored indirect access files. (One PRU is 64 words or 640 six-bit bytes.)

AW = Access word. If the last digit is 4 or greater, the user is allowed to create direct access files.

The following limits are not in effect for version 2.1.1 of the KRONOS operating system but may be in effect in later versions:

NP = Number of local files allowed. This includes active APLUM files and coded files. Allow one extra file when saving or loading a workspace.

MS = Maximum number of mass storage tracks allowed for files, including active copies of indirect access files. One track may be anywhere from 1024 words to 54,784 words depending on the storage device used. Contact installation personnel for details.

In some cases SYSTEM may be printed as one of the above limits, which indicates that there is no limitation on the individual account number, but there may be a general limitation on all timesharing users. Contact installation personnel to determine this limitation.

SETTL, number

Sets the CPU time limit to number. This can be used before entering APLUM to prevent a *TIME LIMIT* error from occurring. The number should be the desired time limit in octal. In order to be meaningful, the time limit should be at least 10 (octal) and the last digit should be a zero. The time limit must not be set to more than the remaining allowance for the session. (You can use the HELLO command to start a new session and get a fresh allotment of CPU time.)

T, number

This command is meaningful only immediately after the system has printed *TIME LIMIT*. The number has the same significance as for the SETTL command. If you type anything other than T, number, a forced exit from APL will occur and the active workspace will be lost. If you have used up the entire CPU time

allotment for the session, hang up the phone and then sign on again and use the RECOVER command.

DISPOSE

The DISPOSE command is used to send a coded file to a batch printer. Note that the first character on each line in the file should be a printer carriage control symbol. A blank is usually used, but a 1 can be used to skip to the next page, or a 0 can be used to skip a line before printing. Numerous other options exist (see KRONOS 2.1 Reference Manual). The following steps would be used to print a file at the central batch site:

```
BATCH
GET,filename
DISPOSE(filename=PR/BC=account)
```

where *filename* is the name of an indirect access coded file and *account* is your account number. If the coded file does not include carriage control characters in the first column, the following procedure should be used instead:

```
BATCH
GET,filename
RETURN,TEMP
COPYSBF,filename,TEMP
DISPOSE(TEMP=PR/BC=account)
```

CHANGE

The CHANGE command can be used to change the name of a file (which includes workspaces), its password, category, or access modes permitted to other users. The following examples show simple forms of the command.

```
CHANGE,newname=oldname
Changes the name from oldname to newname.
```

```
CHANGE,filename/CT=category
Changes the category. The category specified may be P for private, S for semiprivate, or PU for public.
```

```
CHANGE,filename/M=mode
Changes the mode. The mode specified may be R for read, W for write, MODIFY for modify, or RM for read-modify. (Other modes exist but are not of interest for APL/M users.)
```

```
CHANGE,filename/PW=password
Sets the file password. The password may consist of 1 to 7 letters or digits.
```

ENQUIRE,J *jobname*

This command can be used to determine the status of a job submitted using the *CSUBMIT* function (discussed in Chapter 9). If the response indicates the job is not in the system, this usually indicates that it has completed or is presently being printed.

PERMIT

The PERMIT command can be used to give another user access to a private file or to specify the permitted access mode for a particular user of a semiprivate file. The form of the command is:

PERMIT,*filename*,*account=mode*,*account=mode*, ...

The mode for each account number determines the type of access allowed. Meaningful modes for APLUM users are R for read, W for write, RM for read-modify, or MODIFY for modify.

CATLIST

The CATLIST command can be used to examine access information about an individual file. The following examples show how to find information not provided by the API. *LIB* function:

CATLIST/LO=F,FN=*filename*

Similar to *LIB 'filename'* but also gives the password and count of the number of accesses.

CATLIST/LO=FP,FN=*filename*

Gives access information for each user who accessed the specified private or semiprivate file. The information printed includes the number of accesses by each user, the access mode allowed for each user, and the date and time of the last access by each user.

Appendix A. Error Messages

The following list describes the APL error messages and their meanings. It should be noted that most of these cause execution to halt (unless `⌈TRAP` is used to intercept the error processing), but function definition mode prints its error messages and then may continue processing.

00: INTERRUPT

This indicates that an interrupt has been received from a terminal or that the overstrike `⌘` has been entered as the first nonblank symbol for quote-quad input.

01: IMPLICIT ERROR

An implicit argument to a primitive function is not defined. The system variable `⌈CPT` is required for the functions `A=B`, `A>B`, `A<B`, `A≠B`, `A≤B`, `A≥B`, `A⊂B`, `A⊃B`, and `⊆B`. The variable `⌈IO` is required for indexing, the axis operator, `A⊆B`, `A⊇B`, `⊆B`, `⊇B`, and `A⊆B`. The variable `⌈WSID` must be defined for `⌈SAVE`. `⌈RI` is required for `A?B` and `?B`, and `⌈PP` is required for monadic format. `⌈ENV` is required for `⌈CR`, `⌈EX`, `⌈FX`, `⌈NC`, `⌈NL`, `⌈STOP`, `⌈TRACE`, `⌈LOCK`, `⌈LTIME`, `⌈NAMES` and `⌈COPY`.

02: SYNTAX ERROR

Incorrectly formed statement. Check to be sure the statement has matched quotes, parentheses, and brackets. A common error is to forget to place an operation symbol between two variables when concatenation is intended (e.g., `(N N)⊆Q` instead of `(N N)⊆Q`). Other causes include failure to provide a right argument to a function, and use of a branch arrow other than at the left end of a statement. Check the state indicator to be sure a local variable or label is not obscuring a function having the same name.

03: DOMAIN ERROR

The argument is not in the domain of the function or is an improper value for a system variable being specified. The following are examples of ways that domain errors can arise

13.5 (an integer is required), `ORIGIN` (the index origin must be 1 or 0), `PREC` (character arguments are not allowed for many operations, even if the argument is empty), `PRECISION` (printing precision must be between 1 and 15). When `PREC` is not defined, zero is used as `PREC` in domain checks. Thus, `11+18 14` would not be allowed because exact integers are required when `PREC` is zero.

04: LENGTH ERROR

Lengths of the arguments to a function are incompatible, or the operation is not defined for arguments of that length.

05: VALUE ERROR

A variable used in an expression has not been assigned a value, a dyadic function has been used without a left argument, the result variable of a function that returns a result was not assigned a value, or a function was used for which there is no current definition. Check the state indicator to see if a local variable has obscured a global variable or function.

06: RANK ERROR

The ranks of the arguments are incompatible or the operation is not defined for an argument of that rank. For example: `1 1 1` (not defined for vectors unless they have one element), `A[1:2]` (if `A` is a vector it has the wrong rank for the index applied), `3 4 5 0` (not defined for ranks greater than 2).

07: INDEX ERROR

Index out of range. For example, if `A` is a three element vector: `A[4]` in 1-origin, `A[3]` in zero origin, or `A[0]` in 1 origin. To find the current origin, display `ORIGIN`.

08: LIMIT ERROR

The operation exceeds limitations of the computer or the APLUM system. Limit errors can result from: attempts to generate a result greater in magnitude than about `1E322`, attempts to execute a line longer than 120 characters (in a function, arguments to the execute function, or entered as input), attempts to produce an array having a rank greater than 75, or an attempt to produce a result requiring more storage than the entire central memory of the computer.

09: LOCKED OBJECT

Attempt to specify a value for a locked variable (label or group). Locked variables can be redefined only by erasing them and then specifying them.

10: NS FULL

Insufficient space remains in the workspace for the operation. Erase unneeded functions and variables to make more space available, or reset `NS` to allow a larger workspace. However, do not set `NS` to a greater field length than you are validated for or you might actually increase the chance of `NS FULL` (see `LIMITS` in Chapter 12). If there is insufficient room for executing system commands, try using system functions

instead. If that fails, try using specification to reduce the space required for larger variables e.g., `*(*)`. Some space can usually be reclaimed by executing a niladic branch (e.g., `→`). If more than one suspension is on the state indicator, use a niladic branch for each suspension.

12: DEFN ERROR

Incorrect request in function definition mode. May result from providing header information other than the function name when reopening the function, use of a function name already in use for another global function or variable, or an illegal display or line editing request. Another cause is an attempt to close definition of a function having an incorrectly formed header or duplication of names used in the header or as labels.

13: PHRASE NOT FOUND

The phrase specified was not found in the line where it was sought. Be sure to specify the correct line number. Display the line to determine the correct phrase.

14: SI DAMAGE

Information on the state indicator has been lost due to changing a pendent function, by altering a function that is suspended more than once, or by changing the number or relative order of local variables in the header or label variables for a suspended function. This message is a warning--no corrective action is required. The pendent or suspended functions on the state indicator that are affected by *SI DAMAGE* are indicated by enclosing brackets. The affected functions cannot be continued, but they remain on the state indicator as long as other suspensions are above them. When the state indicator collapses to the affected suspension, the system automatically removes that suspension.

15: NAME NOT FOUND

No function or variable having that name exists.

16: NAME IN USE

A function or variable already has that name.

18: MIXED FUNCTION

A mixed function has been used where a dyadic scalar function is required as an argument to an operator. For example: `A+.1B`, `A*.1B`, `Φ/B`.

19: UNDEFINED FUNCTION

No such primitive function exists. For example: `*B`, `*B` (no monadic `*` function).

20: operating system error message

This message is a message from the KRONOS operating system and usually concerns some sort of operation with a file or with a workspace. See the list of common errors under OPERATING SYSTEM ERROR MESSAGES below.

21: FILE DAMAGED

Usually indicates that one record of the file has been damaged. If an attempt to tie the file causes this message, the entire file may have been damaged. Most installations periodically copy all files to tape, and files can be restored to their condition when the last copy was made. Contact installation personnel for assistance. File damage may be reported erroneously when reading a direct access file in RM mode if repeated interference is encountered from another user writing the same record.

22: WRONG TYPE FILE

An attempt was made to use CFREAD, CFPOS, CSUBMIT, or CEWRITE on an APLUM file, or an attempt was made to use PREAD or FWRITE on a coded file. Note that the KRONOS COPY commands do not preserve the type with a copy made from an APLUM file. This error also occurs if an attempt is made to alter a direct access file that was tied in read or read-modify mode.

23: FILE TIE ERROR

An attempt was made to use a file number or file name that was already in use, or an attempt was made to perform an operation (e.g., PREAD, FWRITE) that requires the file to be tied.

OPERATING SYSTEM ERROR MESSAGES

20: filename BUSY

The specified direct access file is tied in an incompatible mode. This may be caused by a system problem or telephone disconnect, in which case the file will be released in 10 minutes or can be accessed by using the KRONOS RECOVER command to resume the session that terminated abnormally. Occasionally a file will be left busy due to an operating system error and will remain busy until a level zero deadstart (usually done at the start of the day). An APLUM direct access file can usually be retrieved from this condition by using RM mode to make a new copy of the file.

20: filename ALREADY PERMANENT

A file having the indicated name already exists. This error may result if a workspace is being saved and a password, category, mode, file type (i.e., IA or DA), or a name different from `WSID` was specified. This error can also occur when `FORSAVE` attempts to create a direct access file having the same name as a file already in existence or when `FUATIE` attempts to store a copy of an indirect access file that was created during the session. If the old file is no longer needed, use `DDPOP` to eliminate it; otherwise, copy the new file to change its name.

20: filename NOT FOUND

The file does not exist under the specified user number, the user is not allowed to access the file, or the user did not provide a correct password for a file requiring a password.

20: ILLEGAL USER ACCESS

The user is either not allowed to create direct access files or is not allowed to create indirect access files.

20: FILE TOO LONG

The file is too large for the limits associated with the account number.

20: PF UTILITY ACTIVE

The computer operations staff is using a permanent file utility program that prevents users from performing operations involving permanent files. Try the operation again.

20: CATALOG OVERFLOW - SIZE

The operation would cause the user's limit on total size of all indirect access files to be exceeded.

20: CATALOG OVERFLOW - FILES

The operation would exceed the limit on the number of files allowed for the account number.

20: PARITY ERROR

20: ADDRESS ERROR

20: DEVICE STATUS ERR.

20: 6581 FUNCTION REJ.

20: DEVICE RESERVED

20: DEVICE NOT READY

Any of these messages indicates a malfunction in the computer or a storage device. Try the operation again, and if the problem persists, notify installation personnel.

20: TRACK LIMIT

There is no space available on the device where the file resides. Be sure you have not accidentally created a gigantic file. If you use very large files, you may need to make special arrangements with the installation personnel.

ABNORMAL EXITS FROM APLUM

PARAMETER ERROR

This error indicates the APLUM command was incorrect in form or that a parameter was specified incorrectly.

TIME LIMIT

A CPU *TIME LIMIT* occurred and the T,number command was not used to continue processing (see Chapter 12).

ABORT

A peripheral processing unit requested that the program be terminated.

OPERATOR DROP

The computer operator intervened and terminated the program.

FILE LIMIT

More active files were used than are allowed by the user's validation limits (see LIMITS in Chapter 12).

SYSTEM ABORT

The operating system terminated the program. This presumably indicates a defect in the operating system.

APL SYSTEM ERROR (or EXCHANGE PACKAGE)

This indicates a defect in the APLUM system or a computer or operating system malfunction. Please report this error to installation personnel along with work that led to the problem and any further output from the APLUM system. Unlike most error messages, this is not an indication of an error by the APL programmer.

OTHER MESSAGES

DZL

This indicates that the input line was cancelled.

OVL

This indicates that the preceding input line was too long for the operating system.

Appendix B. Output Format

Character output is sent to a terminal unaltered except for character translation required for the particular type of terminal and omission of trailing blanks in rows of a matrix. This omission of trailing blanks in character output speeds the printing of the result from `DCR`, the printing of tables of names, and so forth.

Numeric output is ordinarily shown in decimal form unless decimal form would not be sufficiently compact. When decimal form is used, up to `DP` significant digits are shown, but trailing zeros beyond the decimal point are omitted, as is the decimal point itself if no digits follow. Numbers with a magnitude less than 1 are shown with a zero before the decimal point (e.g., 0.025, 0.125). All numbers in a column have their decimal points aligned.

Exponential form is used if decimal form would require more than 3 zeros after the decimal point before the first significant digit, if aligning decimal points in the column would require more than $1.5 \times DP$ characters positions, or if more than `DP` digits would appear to the left of the decimal point. If any number in a column requires exponential format, the entire column is shown in exponential format with the decimal points and exponents aligned. All numbers in the column are shown with the same number of digits in the mantissa. The number of mantissa digits is less than `DP` according to how many trailing zeros would otherwise appear in all numbers in the column. If no numbers in the column have digits beyond the decimal point, the decimal point is omitted.

Numbers in adjacent columns are separated by at least one space. However, no more spaces than necessary are used.

Appendix C. Character Sets and Terminals

Many different types of terminals can be used with the APLUM system. In addition, card readers, printers, and files can be used for input and output. The characters available on these various devices are shown in the table at the end of this section. Many of these devices cannot print the full set of APL characters. APL characters are translated so as to print the same whenever possible. In some cases characters having a similar appearance are substituted (e.g., " for and @ for e). In the table where two characters appear in the same column, either character may be used for input, but all output uses the second character. Note that the APLUM system assumes the same terminal type for input and output. Where there is a blank entry in the table, the "bad character symbol," [A\220], is used. Note that the [A\ indices are for 0-origin. The first 128 elements of [A\, when used in output, cause APLUM to issue the corresponding ASCII code. However, further translation may occur within the operating system for some types of terminals.

TERMINAL CONTROLS

Table C.1 shows the characters used to cancel an input line, correct an input line, stop a program not requesting input (called an "interrupt" elsewhere in the text), and to stop a program requesting input. The two entries for halting a program requesting input are for quote-quad and quad input, respectively.

TERMINAL TYPES

From the point of view of the user, the three APL terminal types are almost identical except for the control functions shown in Table C.1. The only other difference is that the two ASCII terminals have the extra symbols {}~~0~~4 which are represented on Selectric terminals as the overstrikes {} = {}+. The Selectric terminals are distinguished from the others in that they are

based on the IBM Selectric print mechanism. These terminals are further divided into correspondence and EBCDIC terminals. The APLUM system allows either correspondence or EBCDIC terminals to be used. (The APLUM system does not need to distinguish between EBCDIC and correspondence types because the operating system compensates for the differences). The Teletype Model 38 is an example of an ASCII-APL terminal, and the CCI 1030 is an example of a bit-pairing terminal. All of these terminals are shown in the character code tables in the column showing APL symbols.

Full-ASCII terminals are characterized by having upper and lower case letters. Because these terminals do not have all the APL symbols, many APL symbols must be represented by a dollar sign followed by two additional symbols.

ASCII terminals include the Teletype Model 33 and Model 35. These terminals also require the use of the dollar sign sequences to represent most APL symbols.

The BATCH column is for ASCII line printers having 63 printing characters. The B501 column is for the CDC 501 printer which differs in a few character positions from ASCII. The 501 printer prints the KRONOS display code symbols.

The file system translates coded file characters using an ASCII coding in the file, regardless of the terminal type specified on the APLUM control card. If the 501 type printer is used instead of an ASCII printer, the APL symbols " _ ! * ' ? , ~ will come out as ~*+v!+<> instead of #''_!<?@^.

TABLE C.1. TERMINAL CONTROLS

	CANCEL INPUT	CORRECT INPUT	STOP PROGRAM	STOP INPUT	*SIGN-ON CHARACTERS
ASCII with APL print	ESC	LINE-FEED BACKSPACE	BREAK	# or +) RETURN
Full ASCII	ESC	LINE-FEED BACKSPACE	BREAK	\$G. or \$GO	RETURN RETURN
ASCII	ESC	LINE-FEED CTRL-H	BREAK	\$G. or \$GO	RETURN RETURN
Selectric APL	ATTN RETURN	ATTN BACKSPACE	ATTN ATTN	# or +	RETURN

*Sign-on characters apply only at the University of Massachusetts installation. These characters should be the first characters typed after dialing the computer. One exception to the above is that bit-pairing ASCII-APL terminals require @RETURN as the sign-on characters.

QWV Index	APL Symbol (Overstrike)	ASCII Symbol (Name)	TTY 33 Symbol	BATCH Printer	B501 Printer	APL Coded Files
0	N (NU)	(NUL)	\$NU	\$NU	\$NU	
1	H (SH)	(SOH)	\$SH	\$SH	\$SH	
2	S (ST)	(STX)	\$ST	\$ST	\$ST	
3	E (ET)	(ETX)	\$ET	\$ET	\$ET	
4	B (EO)	(EOT)	\$EO	\$EO	\$EO	
5	B (EN)	(ENQ)	\$EN	\$EN	\$EN	
6	K (AK)	(ACK)	\$AK	\$AK	\$AK	
7	B (BL)	(BEL)	\$BL	\$BL	\$BL	
8	B (BJ)	(BS)	\$BJ	\$BJ	\$BJ	
9	H (HT)	(HT)	\$HT	\$HT	\$HT	
10	Z (LF)	(LF)	\$LF	\$LF	\$LF	
11	V (VT)	(VT)	\$VT	\$VT	\$VT	
12	D (FD)	(FD)	\$FD	\$FD	\$FD	
13	H (CR)	(CR)	\$CR	\$CR	\$CR	
14	S (SO)	(SO)	\$SO	\$SO	\$SO	
15	S (SI)	(SI)	\$SI	\$SI	\$SI	
16	B (DE)	(DLE)	\$DE	\$DE	\$DE	
17	D (D1)	(DC1)	\$D1	\$D1	\$D1	
18	D (D2)	(DC2)	\$D2	\$D2	\$D2	
19	D (D3)	(DC3)	\$D3	\$D3	\$D3	
20	D (D4)	(DC4)	\$D4	\$D4	\$D4	
21	K (NK)	(NAK)	\$NK	\$NK	\$NK	
22	S (SY)	(SYN)	\$SY	\$SY	\$SY	
23	B (EB)	(ETB)	\$EB	\$EB	\$EB	
24	S (CA)	(CAN)	\$CA	\$CA	\$CA	
25	B (EM)	(EM)	\$EM	\$EM	\$EM	
26	B (SB)	(SUB)	\$SB	\$SB	\$SB	
27	B (ES)	(ESC)	\$ES	\$ES	\$ES	
28	B (FS)	(FS)	\$FS	\$FS	\$FS	
29	B (GS)	(GS)	\$GS	\$GS	\$GS	
30	B (RS)	(RS)	\$RS	\$RS	\$RS	
31	B (US)	(US)	\$US	\$US	\$US	
32	blank	blank	blank	blank	blank	blank
33	! (!.)	!	\$EX !	\$EX !	\$EX	!
34						
35						
36	\$ (S)	\$	\$DO	\$DO	\$DO	\$
37						
38						
39	'	'	\$QT '	\$QT '	\$QT	'
40	(({	{	{	{
41))	}	}	}	}
42	*	*	*	*	*	*
43	+	+	+	+	+	+
44	-	-	-	-	-	-
45	-	-	-	-	-	-

DA7 Index	APL Symbol (Overstrike)	ASCII Symbol (Name)	TTY33 Symbol	BATCH Printer	B531 Printer	APL Coded Files
46
47	/	/	/	/	/	/
48	0	0	0	0	0	0
49	1	1	1	1	1	1
50	2	2	2	2	2	2
51	3	3	3	3	3	3
52	4	4	4	4	4	4
53	5	5	5	5	5	5
54	6	6	6	6	6	6
55	7	7	7	7	7	7
56	8	8	8	8	8	8
57	9	9	9	9	9	9
58	:	:	\$CL :	\$CL :	\$CL :	:
59	;	;	\$SC ;	\$SC ;	\$SC ;	;
60	<	<	\$LT <	\$LT <	\$LT <	<
61	=	=	\$EQ =	\$EQ =	\$EQ =	=
62	>	>	\$GT >	\$GT >	\$GT >	>
63	?	?	\$QU ?	\$QU ?	\$QU ?	?
64						
65	A	A	A	A	A	A
66	B	B	B	B	B	B
67	C	C	C	C	C	C
68	D	D	D	D	D	D
69	E	E	E	E	E	E
70	F	F	F	F	F	F
71	G	G	G	G	G	G
72	H	H	H	H	H	H
73	I	I	I	I	I	I
74	J	J	J	J	J	J
75	K	K	K	K	K	K
76	L	L	L	L	L	L
77	M	M	M	M	M	M
78	N	N	N	N	N	N
79	O	O	O	O	O	O
80	P	P	P	P	P	P
81	Q	Q	Q	Q	Q	Q
82	R	R	R	R	R	R
83	S	S	S	S	S	S
84	T	T	T	T	T	T
85	U	U	U	U	U	U
86	V	V	V	V	V	V
87	W	W	W	W	W	W
88	X	X	X	X	X	X
89	Y	Y	Y	Y	Y	Y
90	Z	Z	Z	Z	Z	Z

DAV Index	APL Symbol (Overstrike)	ASCII Symbol (Name)	TTY33 Symbol	BATCH Printer	B501 Printer	APL CoCed Files
91	[[\$OB [\$OB [\$OB [[
92	\	\	\$BS \	\$DS \	\$BS \	\
93]]	\$CB]	\$CB]	\$CB]]
94						
95	-	-	\$UL	\$UL -	\$UL -	-
96						
97	A (A_)	a	\$AA	\$AA	\$AA	A
98	B (B_)	b	\$BB	\$BB	\$BB	B
99	C (C_)	c	\$CC	\$CC	\$CC	C
100	D (D_)	d	\$DD	\$DD	\$DD	D
101	E (E_)	e	\$EE	\$EE	\$EE	E
102	F (F_)	f	\$FF	\$FF	\$FF	F
103	G (G_)	g	\$GG	\$GG	\$GG	G
104	H (H_)	h	\$HH	\$HH	\$HH	H
105	I (I_)	i	\$II	\$II	\$II	I
106	J (J_)	j	\$JJ	\$JJ	\$JJ	J
107	K (K_)	k	\$KK	\$KK	\$KK	K
108	L (L_)	l	\$LL	\$LL	\$LL	L
109	M (M_)	m	\$MM	\$MM	\$MM	M
110	N (N_)	n	\$NN	\$NN	\$NN	N
111	O (O_)	o	\$OO	\$OO	\$OO	O
112	P (P_)	p	\$PP	\$PP	\$PP	P
113	Q (Q_)	q	\$QQ	\$QQ	\$QQ	Q
114	R (R_)	r	\$RR	\$RR	\$RR	R
115	S (S_)	s	\$SS	\$SS	\$SS	S
116	T (T_)	t	\$TT	\$TT	\$TT	T
117	U (U_)	u	\$UU	\$UU	\$UU	U
118	V (V_)	v	\$VV	\$VV	\$VV	V
119	W (W_)	w	\$WW	\$WW	\$WW	W
120	X (X_)	x	\$XX	\$XX	\$XX	X
121	Y (Y_)	y	\$YY	\$YY	\$YY	Y
122	Z (Z_)	z	\$ZZ	\$ZZ	\$ZZ	Z
123	[([-)		\$LB	\$LB	\$LB	
124		,	\$MD	\$MD	\$MD	
125) (]-)		\$RB	\$RB	\$RB	
126	~	^	\$TL	\$TL ^	\$TL ^	^
127	B (DZ)	(DEL)	\$DZ	\$DZ	\$DZ	
128		(K0)				
129		(K1)				
130		(K2)				
131		(K3)				
132		(K4)				
133		(K5)				
134		(K6)				
135		(K7)				

A. Index	AFI Symbol (Overstrike)	ASCII Symbol (Name)	TTY33 Symbol	BATCH Printer	B501 Printer	APL Coded Files
136		(K8)				
137		(K9)				
138		(K10)				
139		(K11)				
140		(K12)				
141		(K13)				
142		(K14)				
143		(K15)				
144		(K16)				
145		(K17)				
146		(K18)				
147		(K19)				
148		(K20)				
149		(K21)				
150		(K22)				
151		(K23)				
152		(K24)				
153		(K25)				
154		(K26)				
155		(K27)				
156		(K28)				
157		(K29)				
158		(K30)				
159		(K31)				
160	^	(N0)	\$AN	\$AN	\$AN ^	
161	v	(N1)	\$OR	\$OR	\$OR v	
162	w (~)	(N2)	\$ND	\$ND	\$ND	
163	x (v~)	(N3)	\$NR	\$NR	\$NR	
164	<	(N4)	\$LE	\$LE	\$LE <	
165	=	(N5)	\$NE	\$NE	\$NE =	
166	≥	(N6)	\$GE	\$GE	\$GE ≥	
167	Δ (Δ)	(N7)	\$UG	\$UG	\$UG	
168	∇ (∇)	(N8)	\$DG	\$DG	\$DG	
169	†	(N9)	\$TA	\$TA	\$TA †	
170	‡	(N10)	\$DR	\$DR	\$DR ‡	
171	+	(N11)	\$IS +	\$IS	\$IS	
172	→	(N12)	\$GO	\$GO	\$GO →	
173	R (R*)	(N13)	\$LP	\$LP	\$LP	
174	W (V~)	(N14)	\$LD	\$LD	\$LD	
175	V	(N15)	\$DL	\$DL	\$DL	
176	L	(N16)	\$MN	\$MN		
177	I	(N17)	\$MX	\$MX		
178	x	(N18)	\$ML &	\$ML &	\$ML	&
179	÷	(N19)	\$DV %	\$DV	\$DV	
180	**	(N20)	\$DQ ' '	\$DQ ' '	\$DQ	' '

AV Index	APL Symbol (Overstrike)	ASCII Symbol (Name)	TTY33 Symbol	BATCH Printer	B501 Printer	APL Coded Files
181	-	(N21)	\$NG #	\$NG #	\$NG -	#
182	Δ	(N22)	\$DT	\$DT	\$DT	
183	→ (+)	(N23)	\$RK	\$RK	\$RK	
184	→ (+)	(N24)	\$LK	\$LK	\$LK	
185	o	(N25)	\$CI	\$CI	\$CI	
186	• (O*)	(N26)	\$LG @	\$LG @	\$LG	@
187	o (O-)	(N27)	\$RU	\$RU	\$RU	
188	φ (O)	(N28)	\$RT	\$RT	\$RT	
189	φ (O\)	(N29)	\$TP	\$TP	\$TP	
190	φ (AV)	(N30)	\$DM	\$DM	\$DM	
191	Δ (Δ-)	(N31)	\$DU	\$DU	\$DU	
192	i	(N32)	\$IO	\$IO	\$IO	
193	p	(N33)	\$RO	\$RO	\$RO	
194	⊥	(N34)	\$BV	\$BV	\$BV	
195	T	(N35)	\$RP	\$RP	\$RP	
196	⊥ (IT)	(N36)	\$IB	\$IB	\$IB	
197	c	(N37)	\$IN	\$IN	\$IN	
198	o	(N38)	\$ID	\$ID	\$ID	
199	n	(N39)	\$IX	\$IX	\$IX	
200	u	(N40)	\$UN	\$UN	\$UN	
201	•	(N41)	\$NL	\$NL	\$NL	
202	□	(N42)	\$OD	\$OD	\$OD	
203	□ (Q*)	(N43)	\$QP	\$QP	\$QP	
204	⋈ (\-)	(N44)	\$BT	\$BT	\$BT	
205	/ (/ -)	(N45)	\$SM	\$SM	\$SM	
206		(N46)				
207	⊞ (Q+)	(N47)	\$XD	\$XD	\$XD	
208	u	(N48)	\$OM	\$OM	\$OM	
209	u	(N49)	\$AL	\$AL	\$AL	
210		(N50)				
211		(N51)				
212		(N52)				
213	e	(N53)	\$EP	\$EP	\$EP	
214	⊥ (I*)	(N54)	\$EV	\$EV	\$EV	
215	⋈ (T*)	(N55)	\$FM	\$FM	\$FM	
216	⋈ (-*)	(N56)	\$CN	\$CN	\$CN	
217		(N57)				
218		(N58)				
219		(N59)				
220	⊞ (VA)	(N60)	\$RC	\$BC	\$BC	
221		(N61)				
222		(N62)				
223		(N63)				
224	o (OUT)	(G0)	\$G.	\$G.	\$G.	
225	o (OU)	(G1)	\$OU	\$OU	\$OU	

251 Index	ASCII Symbol (Overstrike)	ASCII Symbol (Name)	TTY 33 Symbol	BATCH Printer	B501 Printer	APL Coded Files
--------------	---------------------------------	---------------------------	------------------	------------------	-----------------	-----------------------

226		(G2)				
227		(G3)				
228		(G4)				
229		(G5)				
230		(G6)				
231		(G7)				
232		(G8)				
233		(G9)				
234		(G10)				
235		(G11)				
236		(G12)				
237		(G13)				
238		(G14)				
239		(G15)				
240		(G16)				
241		(G17)				
242		(G18)				
243		(G19)				
244		(G20)				
245		(G21)				
246		(G22)				
247		(G23)				
248		(G24)				
249		(G25)				
250		(G26)				
251		(G27)				
252		(G28)				
253		(G29)				
254		(G30)				
255		(E0)				

Appendix D. APLUM Control Card

The optional parameters on the APLUM timesharing command (or batch control card) allow specification of the type of terminal (or batch options) to be used, the workspace to be used (thus avoiding a subsequent *LOAD* command), and the constraints on the field length to be used. The general form for the control card is:

APLUM,option=value,option=value ... option=value

Indicating terminal type. When no terminal type is specified, APLUM assumes that an ASCII-APL terminal is being used if the job was entered from timesharing. If the job is a batch or remote batch job, APL assumes that 63 ASCII characters must be used for output (e.g., *i* becomes \$10, *p* becomes \$R0). Other terminal types can be specified as follows:

- TT=COR Correspondence (or EBCDIC) Selectric terminal (DTT=1).
- TT=TYPE Type-pairing terminals (DTT=2).
- TT=BIT Bit pairing terminal (DTT=3).
- TT=ASCAPL For ASCII-APL terminals (including Model 33 Teletype) equipped to print the APL character set. This type is normally assumed for timesharing users (DTT=4).
- TT=TTY33 For Teletype Model 33 terminal or similar devices (DTT=5).
- TT=ASCII For full ASCII terminals not equipped to print the APL character set (DTT=6).

TT=BATCH For devices that support the 63 ASCII character set (TT=7). Usually used for batch or remote batch ASCII printers.

TT=3501 For batch 501 printer (TT=8).

TT=TTY383 Teletype Model 38, keyboard arrangement 3 (TT=9).

At the University of Massachusetts (and at a few other installations), the TT= option should not be specified for any terminals that print the APL character set because the operating system compensates for any differences from ASCII-APL.

Indicating batch output options. The following options are intended primarily for batch users of APL. If the APLUM control card does not specify output options, it is assumed that timesharing users do not wish these options and that batch users do want them.

LO=EPS Any or all of the options E, P, or S may be specified. Any options not specified are not used.

E Echo input. The APL lines read as input are also sent as output.

P Prohibit prompt. The normal APL input prompts (6 spaces or []: plus transparent mode control bytes, a lack of which may cause the input translation for terminals to be incorrect) are not sent to the output file.

S Shift output. Causes a blank to be added to the front of each output line to prevent the first character from being used for printer carriage control.

LO=0 To select none of the E, P, or S options.

Input and output file specification. The input and output files normally used for APL are named INPUT and OUTPUT. For timesharing jobs this causes input to come from the terminal and output to be sent to the terminal. For batch jobs input ordinarily is from the card deck or CSUBMIT file, and output is to a line printer. Other KRONOS files can be used instead. APL translation of input and output is according to the TT option (or the default which depends on whether the job is batch type or timesharing type).

I=file-name Causes input to be read from the named file.

L=file-name Causes output to go to the named file.

L=0 No APL output is produced. (All output is discarded.)

Initial workspace specification. If no workspace is specified, a clear workspace is used. Some effort can be saved by specifying the initial workspace name on the APLUM control card.

WS=wsname APL operations begin with a copy of the named workspace as the active workspace.

UN=user-number Used to specify the user number of the initial workspace. Required only if the user number of the workspace differs from that used when signing on.

PW=password If the workspace belongs to another user and has a password, the password must be provided in order to use it.

Field length specification. If no field length is specified, the APLUM system chooses a minimum field length that depends on the current version of APLUM, and a maximum field length of 24576 words (60000 octal). The field length is used for the APL system and the active workspace. The actual field length used varies dynamically. If storage requirements exceed the maximum field length, a WS FULL message results. Note that specifying a field length greater than the user is validated to use or greater than the limit set by the computer operator may actually increase the chance of a WS FULL error. To check user validation, use the LIMITS control card or the LIMITS command (see Chapter 12).

MX=number Sets the maximum field length. The number is assumed to be in decimal form unless followed immediately by D, in which case it is interpreted as octal. The value is actually rounded up to a multiple of 64.

MN=number Sets the minimum field length. The number is assumed to be in decimal form unless followed immediately by E, in which case it is interpreted as octal. The value is actually rounded up to a multiple of 64.

Appendix E. Numerical Limits and Precision

The CYBER computers can represent nonzero numbers having magnitudes in the approximate range $1.27E322$ to $1.27E-322$. An operation that would ordinarily produce a number smaller in magnitude than $1.27E322$ actually produces zero. Most operations produce a *LIMIT ERROR* when the result would exceed $1.27E322$ in magnitude; however, simple operations such as addition and multiplication can produce $+\infty$ or $-\infty$ which are printed as $1.99E999$ and $-9.99E999$. Using these infinite values for any other operations will result in a *LIMIT ERROR*.

Numbers within this magnitude range are represented with an accuracy of about 14 decimal digits (more precisely, to within 1 part in 2^{48}). The simple operations such as addition, subtraction, multiplication, and division can be expected to be accurate to within 1 part in 2^{48} except when cancellation magnifies the errors. However, operations involving numbers that are integers or powers of 2 give exact results unless the magnitudes differ greatly. For example, exact results are given by: $.5+4$, $.25 \cdot 128$, $8-3$.

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